

Chapter 5

γ -ray Studies

In the first week you will measure total cross-sections for various materials and become familiar with the apparatus. In the second week you will study Compton scattering.

5.1 Objectives of the Experiment:

- To appreciate the role of γ -rays in nuclear electromagnetic transitions.
- To become familiar with the experimental methods of γ -ray spectroscopy, starting with the fundamental mechanisms which occur when a γ -ray enters a detector such as a NaI(Tl) crystal (sodium iodide doped with thallium), and concluding with an understanding of a Multi-Channel analyzer (MCA), which processes amplified signals from the detector.
- To investigate quantitatively the passage of γ -rays through matter.

There are two dominant scattering processes for γ -rays of the energies we will use here. These are Compton scattering in which the γ -ray scatters from an essentially free electron and the photo-effect in which the γ -ray is absorbed by an atom which then ejects an electron with all the available energy. These are discussed below.

5.2 Compton Scattering

Klein and Nishina[?] derived the following equation for Compton scattering. The Klein-Nishina cross-section is:

$$\frac{d\sigma}{d\Omega} = \frac{r_c^2}{2} \left(\frac{E'_\gamma}{E_\gamma} \right)^2 \left(\frac{E_\gamma}{E'_\gamma} + \frac{E'_\gamma}{E_\gamma} - \sin^2 \theta \right) \quad (5.1)$$

Here, E_γ is the incident photon's energy, $E_{\gamma'}$ is the outgoing photon's energy, and $r_c = e^2/mc^2$ is the classical radius of the electron of mass m . One can solve the relativistic

kinematic equations for $E_{\gamma'}$, in terms of E_{γ} and θ , the scattering angle for the photon to obtain:

$$E_{\gamma'} = \frac{E_{\gamma}}{1 + \epsilon(1 - \cos \theta)} \quad (5.2)$$

where $\epsilon = E_{\gamma}/mc^2$. Putting this into Eq.5.1 one obtains:

$$\frac{d\sigma}{d\Omega} = \frac{r_c^2(1 + \cos^2 \theta)}{2} \left\{ \frac{1}{[1 + \epsilon(1 - \cos \theta)]^2} \left[1 + \frac{\epsilon^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \epsilon(1 - \cos \theta)]} \right] \right\} \quad (5.3)$$

Now one would use Feynman diagrams to calculate this cross-section. Klein and Nishina did not, since they had not yet been invented. The lowest order Feynman diagram for Compton scattering is shown in Fig. 5.1

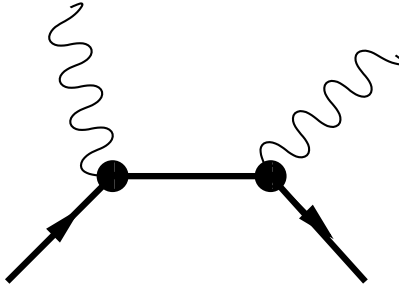


Figure 5.1: Feynman diagram for Compton Scattering. A photon, represented by a wiggly line interacts with an electron, this in turn radiates a photon. A more detailed discussion will have to wait until graduate school.

5.2.1 Photoelectric Cross-section

In *Alpha-, Beta- and Gamma-ray spectroscopy* edited by K. Siegbahn [?] the dominant K-shell cross-section is given as:

$$\sigma_{pe} = 1.367 \times 10^{-22} (\alpha Z)^5 \frac{1}{\epsilon} \text{cm}^2/\text{atom} \quad (5.4)$$

Note that $\alpha \approx 1/137$ is the “fine structure constant” and $\epsilon = E_{\gamma}/mc^2$ the ratio of the incoming γ -ray’s energy to the rest mass energy of an electron. For aluminum σ_{pe} is only about 1% of the Compton cross-section and therefore will be ignored in the Compton scattering part of this lab. For lead with $Z=82$ the photo-effect cannot be ignored.

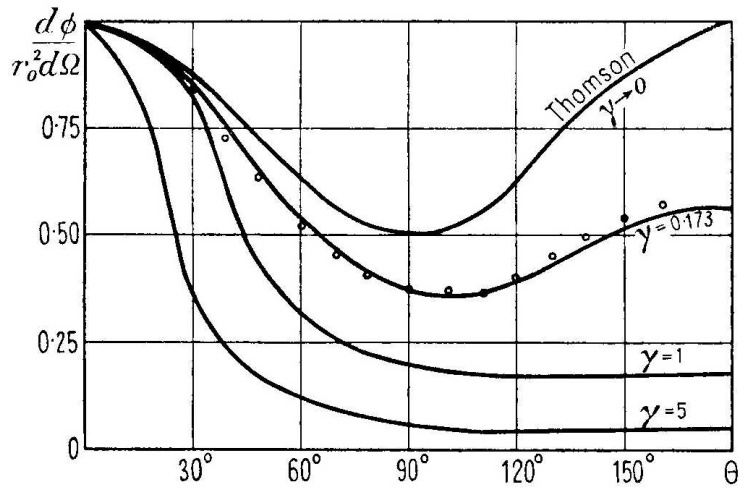


Figure 5.2: Theoretical and experimental differential cross-sections. Taken from Heitler[?]

5.3 Equipment

The general layout is shown in Fig. 5.5. You will use a set of weakly radioactive sources: ^{22}Na , ^{137}Cs , and ^{60}Co . See Fig. 5.4 for the energy levels.

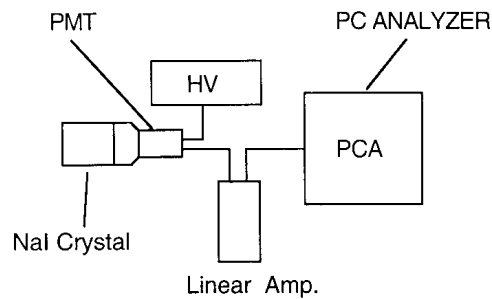


Figure 5.3: A NaI(Tl) detector

5.4 Procedure: First Week

Signals Determine the amplitudes, rise-time and fall time of the signals directly from the photo-multiplier tube. You may need a 50 ohm terminator at the input to the oscilloscope. Why? .

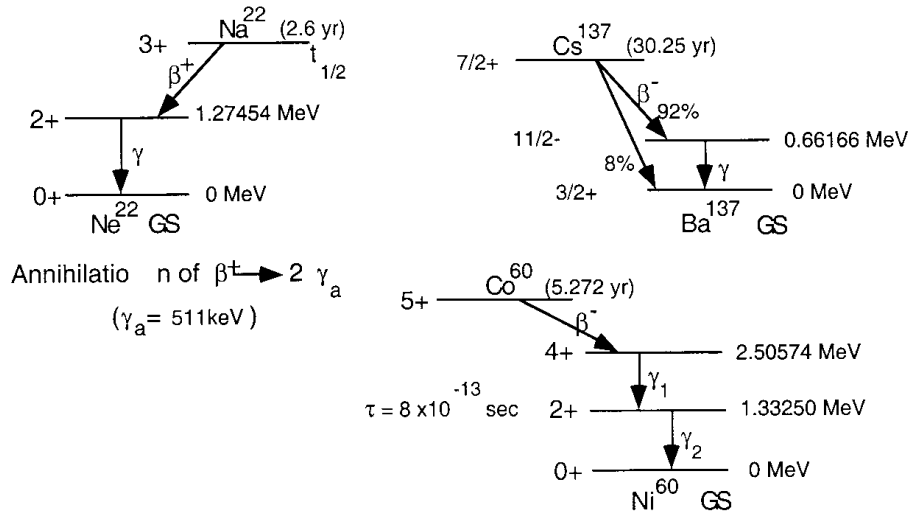


Figure 5.4: Nuclear states and their decay characteristics.

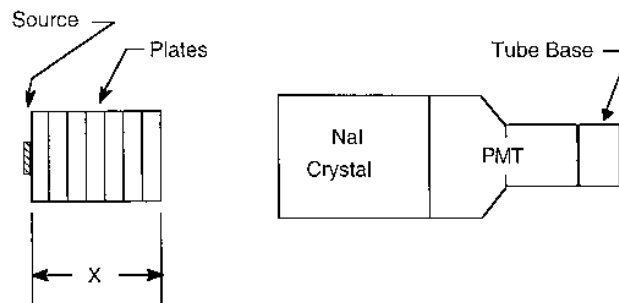


Figure 5.5: γ -rays from the source are scattered by the atoms in the plates.

Calibration Determine the photo-peak channel numbers for the two peaks of ^{60}Co , and ^{22}Na , and the single peak of Cs. Use the known energies from Fig. 5.4 and make a plot of E versus channel number. Fit this to a straight line: $E = a + b \cdot n$. Determine the constants a and b and their errors. You will need this calibration for later use.

Compton Edges Determine the channel number for the Compton edges (see Fig. 5.3). From these and your calibration determine the γ -ray energies of these edges. Compare to the theoretical values.

Line Widths Determine the full width at half maximum, fwhm, for each peak. You can use the region of interest feature to do this. Make a plot of fwhm vs. \sqrt{E} for all the peaks. Assume the energy measured is proportional to some number N, and the fwhm is proportional to \sqrt{N} , both with the same proportionality constant. Determine N for

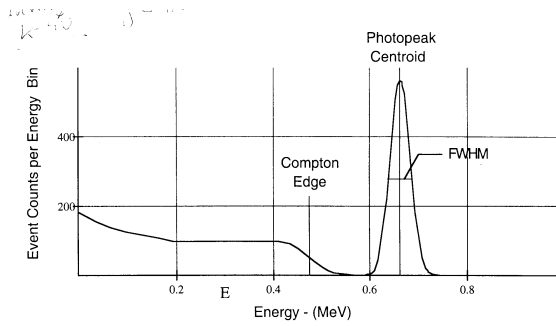


Figure 5.6: Pulse height distribution for a gamma-ray incident on a NaI(Tl) detector. Note the Compton edge, and the width of the photo-peak.

one of the peaks which you identify. This \sqrt{N} behavior is characteristic of Poisson statistics which is appropriate for counting numbers.

Absorption Cross-sections For ^{137}Cs , ^{22}Na and ^{60}Co place varying thickness of plastic, aluminum, copper, and lead between the sources and the detector. Does the position of the photopeak change with absorber thickness? Plot the counts in the photopeaks versus absorber thickness. From these determine the absorption cross-sections.

5.5 Measurements: Second Week

- Check the energy calibration
- Determine the number of counts for fixed live-time as a function of angle with the aluminum cylinder in place. You might wish to take more time at the back angles.
- Repeat the previous with the target cylinder removed. You might want to try several shielding configurations with both sets of measurements. Record the shielding arrangement that you use.
- Determine the peak energy as a function of angle. Compare this to the expected values for E_c from:

$$\frac{1}{E_c} = \frac{1}{E} + \frac{1}{mc^2}[1 - \cos \theta] \quad (5.5)$$

, which can be obtained from energy and momentum conservation.

- Determine $\frac{d\sigma(\theta)}{d\Omega}$. Plot this as a function of angle and plot the theoretical $\frac{d\sigma(\theta)}{d\Omega}$. This will take a fair amount of work. Describe and try to explain the discrepancies.

5.6 Considerations

We are using the apparatus shown in Fig. 5.7. With some dimensions shown in Fig. 5.8. We divide our considerations into:

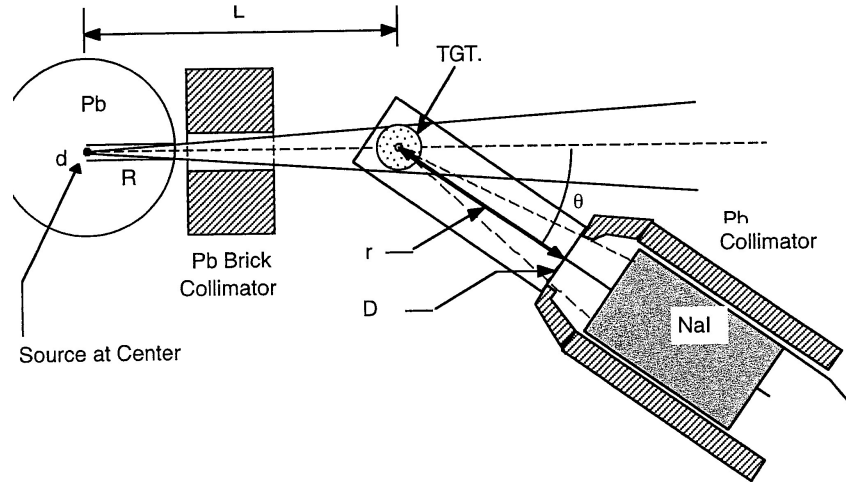


Figure 5.7: The experimental apparatus.

- Beam Characteristics
- Target Considerations
- Detector Considerations

5.6.1 Beam

We start by determining the number of decays per second of ^{137}Cs there are. The activity N_{0s} at a certain time in the past is given on the source. Note that a Curie is: 3.7×10^{10} decays per second. Presuming the half life is $\tau_{1/2}$, the current activity is:

$$N_s = N_{0s} \cdot \left(\frac{1}{2}\right)^{t/\tau_{1/2}} \quad (5.6)$$

The number of γ -rays per second and steradian is then:

$$\frac{dN_s}{d\omega} = \frac{N_s}{4\pi} \quad (5.7)$$

The beam is limited by the aperture shown in Fig. 5.8. Even here there are approximations. The beam is not completely and cleanly collimated since the γ -rays will penetrate a little into the collimator and can then scatter back into the beam.

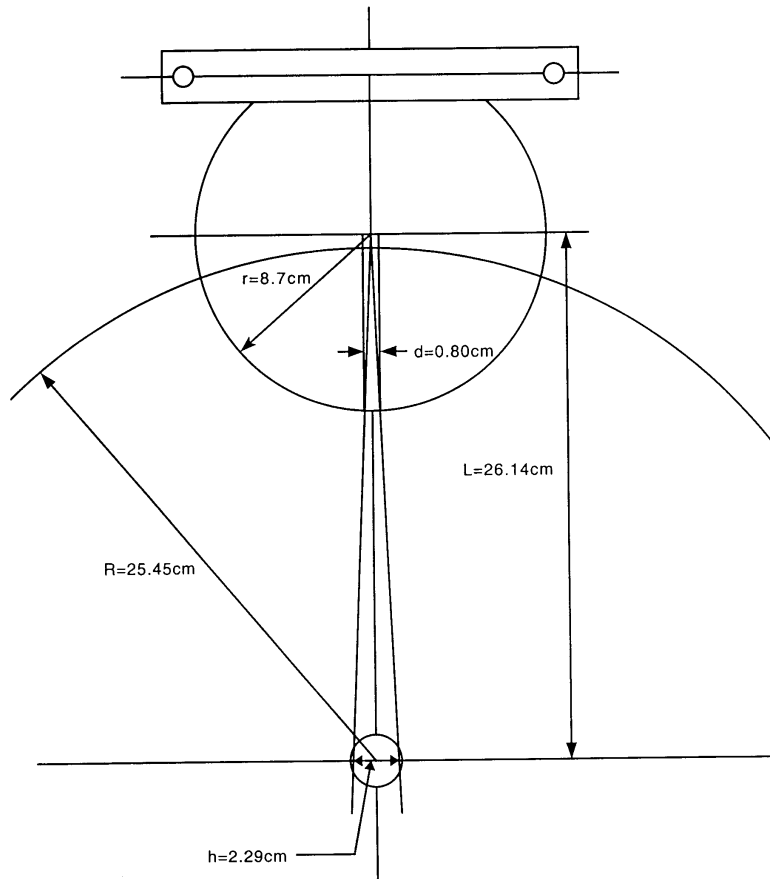


Figure 5.8: The experimental apparatus showing dimensions.

5.6.2 Target

Here we consider the probability of scattering into some solid angle. If we break the beam into small cones of solid angle $d\omega$, see Fig. 5.9, which intersect the target and have a path of length $l(\vec{\omega})$, then the number of scatterers is approximately $\rho l(\vec{\omega})R^2 d\omega$, where ρ is the volume density of scatterers and R is the distance from the source to the target center.

The fraction of the incident particles into $d\omega$ that scatter into $d\Omega$ is then:

$$f = \frac{dN_s(\vec{\omega})}{d\omega} d\omega \cdot \frac{l(\vec{\omega})R^2 d\omega \rho}{R^2 d\omega} \frac{d\sigma}{d\Omega} \quad (5.8)$$

which, after integrating over the target ($d\omega$), is approximately:

$$\frac{dN_d}{d\Omega} = \frac{N_s}{4\pi R^2} 0.86D\pi \left(\frac{D}{2}\right)^2 \frac{d\sigma}{d\Omega} \quad (5.9)$$

where D is the diameter of the target rod and N_d is the number into the detector..

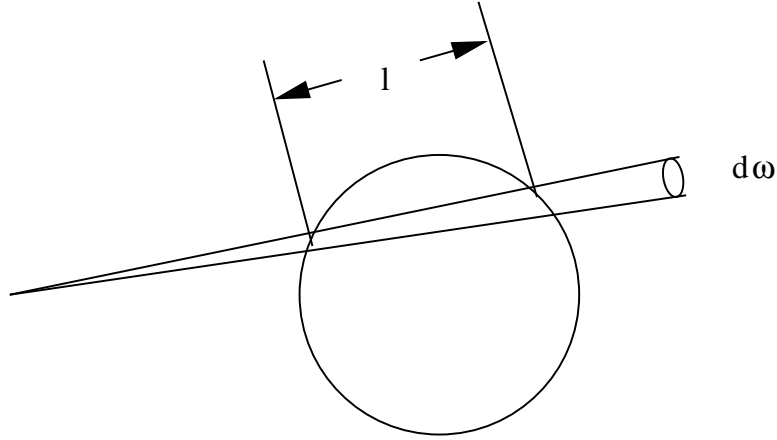


Figure 5.9: The target geometry.

5.6.3 Detector

One needs to determine the solid angle, $\delta\Omega$ of the aperture of the detector, the total probability of detection of a γ -ray of a particular energy (obtained from Fig. 5.10), $P_{tot}(E)$ and if one only counts the photo-peak, the fraction of the spectrum which is in the photo-peak: $f_{pp} = P_{pp}/P_{tot}$ (obtained from Fig. 5.11). The detected counts are then:

$$N_{det} = \frac{dN_d}{d\Omega} \delta\Omega P_{tot} f_{pp} \quad (5.10)$$

5.6.4 Total Cross-section

Consider the situation depicted in Fig. 5.12. If the target is very thin we can assume that most of the particles go through and are then detected. Let N_{inc} be the number incident on the target and N_{det} be the number detected. If the target is not present then we assume $N_{det} = N_{inc}$. If the target is very thin $N_{det} \approx N_{inc}$ and the probability that a scattering has occurred is:

$$P = \frac{N_{inc} - N_{det}}{N_{inc}} = 1 - \frac{N_{det}}{N_{inc}} \quad (5.11)$$

The fraction, f , of this area of the target taken up by scatterers is:

$$f = N_{tgs}(cm^{-2}) \cdot \sigma(cm^{-2}) \quad (5.12)$$

where $N_{tgs}(cm^{-2})$ is the number of scatterers per cm^2 , and σ is the area taken up by each scatterer. Evidently:

$$P = f \quad (5.13)$$

Absorption Efficiency of NaI

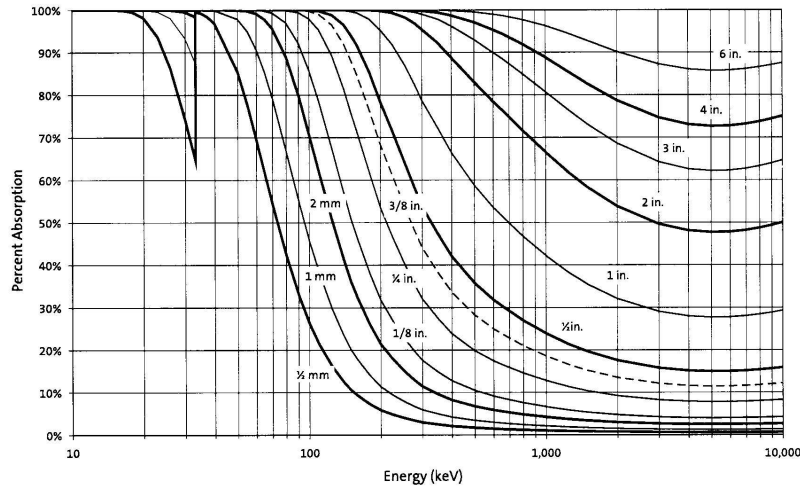


Figure 5.10: Absorption in NaI

So:

$$1 - \frac{N_{det}}{N_{inc}} = N_{tgts}(cm^{-2}) \cdot \sigma(cm^2) \quad (5.14)$$

$$\sigma(cm^2) = \frac{1 - \frac{N_{det}}{N_{inc}}}{N_{tgts}(cm^{-2})} \quad (5.15)$$

Now suppose one has a detector which will detect any scattering that occurs, but not particles which don't scatter. then:

$$\sigma(cm^2) = \frac{N_{dets}}{N_{inc}N_{tgts}(cm^{-2})} \quad (5.16)$$

5.7 Differential Cross-section

In general the scattering will be different to different angles. Consider Fig. 5.13.

We may define $\frac{d\sigma}{d\Omega}\delta\Omega$ to be a small area associated with each scatterer which leads to a particle scattered into the detector. With this definition:

$$\frac{d\sigma}{d\Omega} = \frac{N_{dets}}{N_{inc}N_{tgts}(cm^{-2})\delta\Omega} \quad (5.17)$$

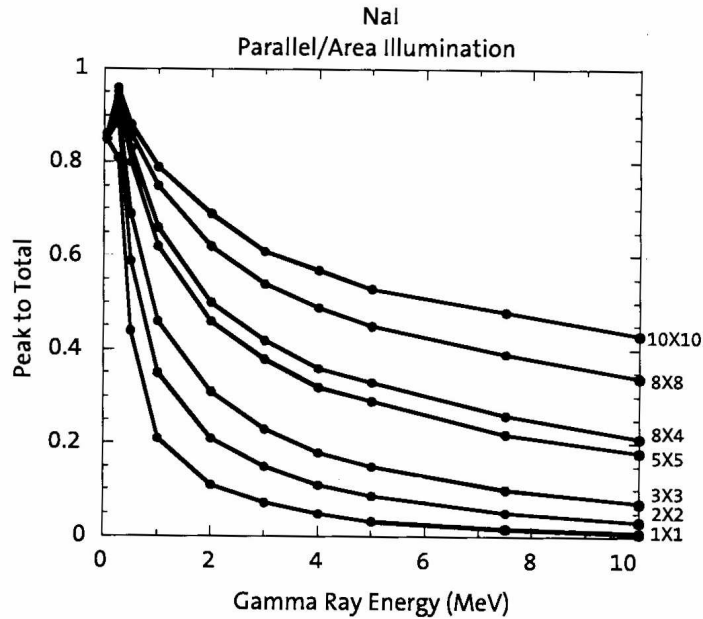


Figure 5.11: Peak to total ratios

5.7.1 The Thomson Cross-section

This cross-section corresponds to the case of a photon scattering from a charge. Usually only scattering from electrons need be considered. A heuristic derivation of the radiated power, P_r , from an accelerated charge will first be given. Then the incoming photon power will be associated with $n\hbar\omega$, where n is the number of incoming photons per second. The outgoing power will be then $n'\hbar\omega$, with the same frequency which is the case for low frequency photon scattering. The Thomson cross-section is:

$$\sigma_T = n'/n = P_r/P_i \tag{5.18}$$

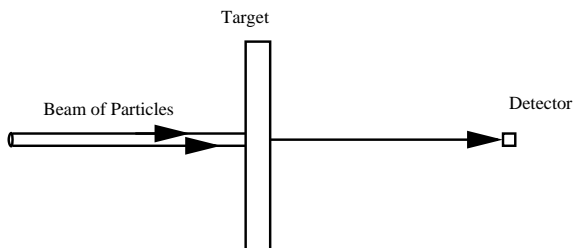


Figure 5.12: An incident beam of particles strikes a target and some scatter. If no scattering occurs it is assumed that the particle will reach the detector and be detected.

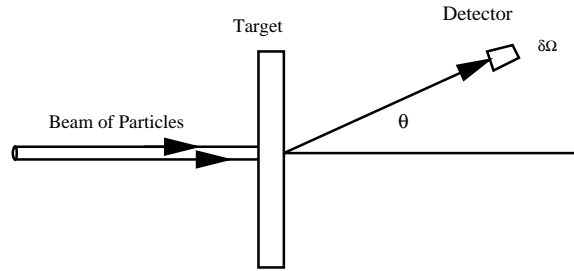


Figure 5.13: An incident beam of particles strikes a target and some scatter into a detector at an angle θ and ϕ subtending a solid angle $d\Omega$.

5.7.2 Heuristic Derivation of P_r

The classical radiated power from an accelerated charge is derived in many Electricity and Magnetism texts, e.g., *Classical Electrodynamics* by J. D. Jackson[?]. What follows is not rigorous, but does yield the right result and can easily be used to re-derive the result.

One knows that the radiated power is proportional to the square of the electric field in the radiation field. This electric field is certainly proportional to e , the charge in question.

One knows that a stationary charge does not radiate. Neither does a constantly moving charge in vacuum. Thus we expect the radiation field to be proportional to a , the acceleration of the charge.

Thus, the radiated power is:

$$P_r \propto e^2 a^2 \quad (5.19)$$

But, power should have units of $e^2/(ls)$. Note that this is in Gaussian units where $\phi = e^2/r$, a set of units it might be useful to learn about. If you want you could re-derive the results here in SI units. In Eq. 5.19 the right hand side has units: $e^2 l^2/t^4$. Thus, to make the right hand side have the right units one needs to divide by $(l/s)^3$. The only thing around with the units of l/t is the speed of light: c . So, now the right hand side is $e^2 a^2/c^3$. We are almost finished deriving the power from an accelerated charge. One needs to note that if the acceleration is along the z axis, one can only observe the motion along the x and y directions, i.e., $2/3$ of the three directions. Thus we are left with the final result:

$$P_r = \frac{2}{3} e^2 a^2 / c^3 \quad (5.20)$$

This is correct, in Gaussian units.

5.7.3 Cross-section

The incoming power is:

$$P_i = \frac{E_i^2}{4\pi}c = n\hbar\omega \quad (5.21)$$

where E_i is the incoming photon's electric field and ω is the photon's angular frequency. The acceleration, $a = eE_i/m$ where m is the mass of radiating charge. So, we can re-write Eq. 5.20 as:

$$P_r = \frac{2}{3} \frac{e^2 E_i^2 e^2}{m^2 c^4} c = \frac{2}{3} \left(\frac{e^2}{mc^2} \right)^2 4\pi P_i \quad (5.22)$$

So, finally we get:

$$\sigma_T = \frac{8\pi}{3} r_c^2 \quad (5.23)$$

where $r_c = e^2/mc^2$, the so-called classical radius of the electron. Approximately, $r_c = 2.8 \cdot 10^{-13}$ cm.

5.8 NaI Detector and Photo-multiplier Tube

A NaI(Tl) scintillator mounted on a photo-multiplier tube is shown in Fig. 5.14. Photons of energy $h\nu_\gamma$ enter the scintillator crystal. Inside this crystal Compton scattering and photo-electric processes occur. In Compton scattering some of the incident photon's energy is transferred to an electron and a lower energy photon. In a photo-electric essentially all of the photon's energy is transferred to an electron. A nucleus takes up the momentum necessary for energy momentum conservation. The Compton scattered lower energy photon may also scatter again further transferring energy to electrons.

The electrons discussed above travel through the NaI crystal exciting the Na and I atoms. These atoms de-excite, emitting photons. These photons excite the Tl atoms, which then de-excite with photons, some of which enter the photo-multiplier tube striking the photo-cathode to produce electrons, for example, N_c . (The photo-cathode is very near the entrance to the tube.) These electrons are accelerated by an applied electric field and strike the first dynode, D1. The dynodes are electrodes held at ever increasing potential. Upon striking the dynode, more than one electron is ejected, say n , are ejected. This acceleration and ejection process is repeated for 10 or so dynodes. Thus the number of electrons which finally reach the anode (the last dynode) is then $N_a = N_c \times n^{10}$. The charge: $e \cdot N_a$ produces a voltage at the SIG output.

5.8.1 Classification of Radionuclear Decays

In 1896 Becquerel made the first observation of radioactivity. Investigations in the early decades of this century soon led to a classification of radioactive materials as α , β , and γ emitters. The process of spontaneous fission was first observed much later.

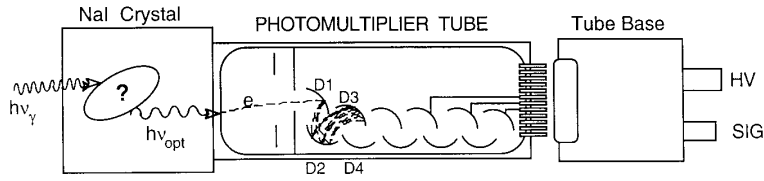


Figure 5.14: Photo-multiplier tube assembly with NaI scintillator.

α **emitters** tend to be isotopes of transuranian nuclei (mass number $A > 208$). Unstable in their nucleon configuration, they decay to a daughter nuclide of mass $(A-4)$ plus an alpha particle. The α “ray” (or particle) was soon identified as a fully ionized helium nucleus which departs the scene of decay with about 5 to 10 MeV of kinetic energy.

β **emitters** tend to be unstable nuclear isotopes which have a low to medium mass number A . The *beta* particle was determined to be an electron (or positron): $n \rightarrow p + e^- + \bar{\nu}$.

γ **emitters** span the full range of the nuclear chart. γ rays from these sources represent the photons emitted in electromagnetic transitions as an excited nucleus makes a transition from an upper to a lower energy level.

Check out the “Chart of the Nuclides” in room Millington 303 to identify α , β , and γ emitters.