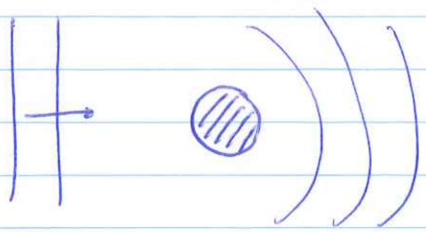
Low energy scattering / s-wave scattering

If the energy of an incoming particle is E (kinetic energy) $E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}}$

Low energy $\longleftrightarrow$ small k

Example from before: Hard sphere

$$V(r) = \begin{cases} \infty & r<a \\ 0 & r>a \end{cases}$$



Previously, we calculated that

$$a_\ell = -j_\ell(ka)/ik\,h_\ell^{(1)}(ka)$$

$$\sigma = \frac{4\pi}{k^2} \sum_\ell (2\ell+1) \left| \frac{j_\ell(ka)}{h_\ell^{(1)}(ka)} \right|^2$$

and for $ka \ll 1$

$$\sigma \approx \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} \frac{1}{2\ell+1} \left[\frac{2^\ell \ell!}{(2\ell)!}\right]^4 (ka)^{4\ell+2}$$

For $ka \ll 1$ the s-wave scattering is the strongest

$$\sigma_s = 4\pi a^2$$

One can show that $\delta_0 = -ka$ (s-wave scattering phase)

$$\sigma_s = \frac{4\pi}{k^2} \sin^2 \delta_0 = \frac{4\pi}{k^2} \sin^2(ka) \approx 4\pi a^2$$

In experiment we usually can change k (energy of the particle)

if $\delta_0 \sim \pi \Rightarrow \sin^2\delta_0 = 0$, $\sigma = 0$ $k \sim \pi/a$

anomolously low scattering (Ramsauer-Townsen effect)

(check the paper!)

In the interest of clarity, only the 0.125, 0.250, 0.500, and 0.750 normalized equipotentials (isotherms) are shown. Again, as in the previous example, it is essential that sufficient electrical contacts be made on the outer conductor boundary. The contact points are shown as black circles in the diagram. The center of the circular silver conductor is located so that it coincides with one of the terminals below the Cenco mapping board to facilitate use of the drawing template.

Numerical computation work was performed on an IBM 1620 computer at the University of Wisconsin–Milwaukee Computer Center while the author was on the staff of the National Science Foundation 1967 Summer Institute there.

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Demonstration of the Ramsauer–Townsend Effect in a Xenon Thyratron

Stephen G. Kukolich

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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The anomalously small scattering of electrons near 1 eV energy by noble gas atoms may be easily demonstrated using a 2D21 xenon thyratron. This experiment is suitable for a lecture demonstration or for an undergraduate physics laboratory. The probability of scattering and the scattering cross section may be obtained as a function of electron energy by measuring the grid and plate currents in the tube.

The scattering cross section for electrons on noble gas atoms exhibits a very small value at electron energies near 1 eV. This cross section is much smaller than that obtained from measurements involving atom–atom collisions. This is the Ramsauer–Townsend effect and provides an example of a phenomenon which requires a quantum mechanical description of the interaction of particles. If the atoms are treated classically as hard spheres, the calculated cross section is independent of the incident electron energy and we cannot account for the Ramsauer–Townsend effect. If the noble-gas atoms are considered to present an attractive potential (e.g., square well, screened Coulomb) of typical atomic dimensions, the solution of the Schrödinger equation for the electrons indicates that the cross section will have a minimum at electron energies near 1 eV. Reviews of the Ramsauer–Townsend effect are given by Mott and Massey[1] and Brode.[2]

[1] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1965), 3rd ed., Chap. 18.

[2] R. B. Brode, Rev. Mod. Phys. **5**, 257 (1933).

The problem of scattering of electrons by a square well is considered in many introductory quantum physics texts.[3–7] The one-dimensional model predicts that the scattering will go to zero whenever half the electron wavelength in the well is a multiple of the well width. The difficulty with this model is that only one distinct minimum is observed.

A slightly better model of the xenon atom is a three dimensional square well. Then the scattering cross section will have a very small value when the phase shift δ_0 of the $l=0$ partial wave is π. Here the scattering due to the $l=0$ partial wave will vanish and the scattering due to higher l partial waves will be small if the width of the

[3] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Co., New York, 1955), Chap. 5.

[4] E. Merzbacher, *Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1955), Chaps. 6, 12.

[5] D. Bohm, *Quantum Theory* (Prentice-Hall Inc., Englewood Cliffs, N.J., 1951), Chaps. 11.9, 21.51.

[6] A. Messiah, *Quantum Mechanics I* (North-Holland Publ. Co., Amsterdam, 1961), Chaps. III-6.

[7] R. M. Eisberg, *Fundamentals of Modern Physics* (John Wiley & Sons, Inc., N.Y., 1961), Chap. 15.

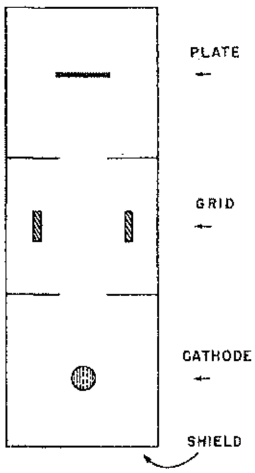


FIG. 1. Cross section of the 2D21 thyratron.

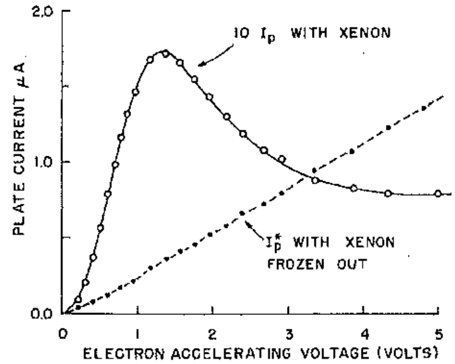


FIG. 3. The plate current I_p as a function of the voltage V, and $I_p{}^*$ the plate current with the xenon frozen out with liquid nitrogen.

well is small.[1] When the $l=0$ phase shift becomes 2π, or higher l phase shifts become π at higher values of electron energy, the dips in the cross section will not be as prominent since contributions from other values of l will not be small. The well parameters may be adjusted to give a minimum at the observed energy. This model predicts the Ramsauer–Townsend effect in a qualitative way, but does not give quantitative agreement over a wide range of electron energies. The results of more accurate calculations with a screened coulomb potential are given by Mott and Massey.[1]

I. THE EXPERIMENT

The 2D21 thyratron is very well suited for a demonstration of the Ramsauer effect. The shield (grid 2) is a boxlike structure with three sections connected by apertures (see Fig. 1). The electron beam originates at the cathode in the first section, passes through the second section, and part of it is collected on the plate in the third section. The xenon pressure in the tube is approximately 0.05 Torr. A diagram of the circuit is shown in Fig. 2. The shield current is proportional to the intensity of the electron beam at the first aperture. After the first aperture the beam passes through an equipotential region where the scattering takes place. In this region the beam intensity is $J=J_0e^{-x/\lambda}$, where λ is the mean free path. If the plate is a distance l from the first aperture, the intensity at the plate is $J_p=J_0e^{-l/\lambda}$ or $J_p=J_0(1-P_s)$, where P_s is the probability of scattering. The plate current is $I_p=I_sf(V)(1-P_s)$, where I_s is

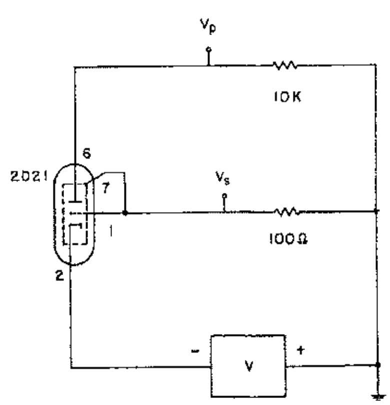


FIG. 2. Diagram of the circuit for the Ramsauer effect experiment. The filament of the 2D21 (pins 3, 4) is heated by 4 V dc.

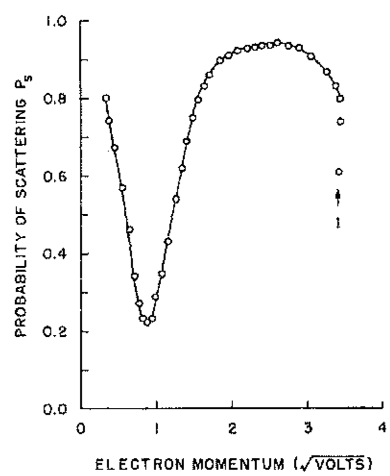


FIG. 4. The probability of scattering P_s as a function of $(V-V_s)^{1/2}$, where $V-V_s$ is the electron energy. Ionization occurs at "I".

the shield current and $f(V)$ is a geometrical factor which contains the ratio of the angle intercepted by the plate to the angle intercepted by the shield and a factor due to space charge effects near the cathode. To measure $f(V)$ we freeze out the xenon by dunking the top of the tube in liquid nitrogen. This reduces the xenon pressure to $\sim 10^{-3}$ Torr and P_s becomes very small so we get $f(V) \cong I_p^*/I_s^*$. Now we have $P_s = 1 - I_p I_s^*/I_s I_p^*$. Figure 3 shows that I_p has a maximum near 1 eV and that I_p/I_p^* approaches one there, indicating that there is very little scattering. At higher energies I_p/I_p^* is very small indicating a large probability of scattering. A plot of P_s calculated from the data using the above equation is shown in Fig. 4.

The probability of scattering is related to the mean free path by the relation $P_s = 1 - e^{-l/\lambda}$. For the 2D21 $l = 0.7$ cm so we can calculate λ. The cross section σ is related to λ by $n\sigma = 1/\lambda$, where n is the number of atoms per unit volume. A plot obtained from our values of P_s is shown in Fig. 5. A similar set of data for P_c ($P_c = P/\lambda$, where P is the pressure in Torr) given by Brode[7] is shown in Fig. 6. In the 2D21 fairly large angular deflections must be produced to scatter an electron out of the beam (greater than ~ 0.2 rad) so the cross section measured in the 2D21 will be smaller than Brode's data.

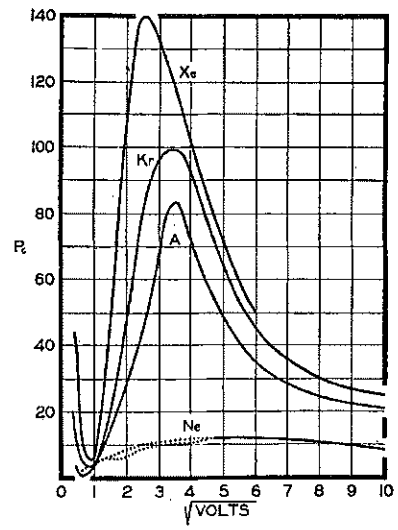


FIG. 6. The probability of collision P_c (= pressure times $n\sigma$) as a function of $(V)^{1/2}$ where V is the electron energy (from Brode see Ref. 2).

II. EXPERIMENTAL DETAILS

The filament of the 2D21 is operated on 4 V dc. This is lower than the recommended value of 6.3 V, but tends to reduce space charge effects. Since the cathode temperature is lower, the thermal kinetic energy of the electrons is smaller and this will result in a narrower distribution of electron energies. The shield and plate currents are obtained by measuring the voltages V_s and V_p with two Keithley model 600A electrometers (see Fig. 2). The voltage source V is a well regulated and filtered supply which may be varied between 0–15 V. The electron energy plotted in the figures is $V - V_s$. We have not included a correction for the contact potential difference between the cathode and the shield. This contact potential difference is approximately 0.4 V and was measured by noting that ionization occurs when $V - V_s$ is 0.4 V less than the tabulated ionization potential. A similar value was obtained by measuring the value of V required to cut off the electron current to the shield. The voltages V_s and V_p range from a few millivolts to a few tenths of a volt. The data may be displayed on an oscilloscope by using an audio oscillator for the source V and for the x axis of the 'scope.

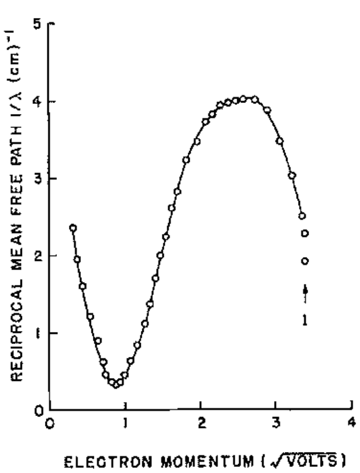


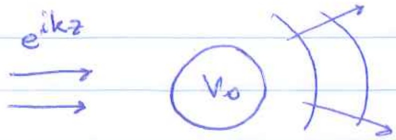
FIG. 5. The cross section times density $n\sigma = 1/\lambda$ as a function of $(V - V_s)^{1/2}$, where $V - V_s$ is the electron energy. Ionization occurs at "I".

ACKNOWLEDGMENT

This experiment was suggested by Professor R. Weiss as a demonstration in an introductory course in quantum physics given by him at MIT.

What if the sphere is not 100% hard?

$$V = \begin{cases} V_0 & \text{for } r<a \\ 0 & r>a \end{cases}$$



This problem is also analytically solvable for $l=0$ (no angular dependence)

$$\delta_0 = \begin{cases} \tan^{-1}\left\{ \dfrac{k}{\sqrt{k^2-\varkappa^2}} \tan\sqrt{k^2-\varkappa^2}\,a \right\} - ka & k>\varkappa \\ & \text{(above barrier)} \\ \tan^{-1}\left\{ \dfrac{k}{\sqrt{\varkappa^2-k^2}} \tanh\sqrt{\varkappa^2-k^2}\,a \right\} - ka & k<\varkappa \\ & \text{(below barrier)} \end{cases}$$

here $\varkappa = \sqrt{2mV_0/\hbar^2}$ - strength of the potential

Note that for $ka \ll 1$ and $k \ll \varkappa$ (low-energy limit)

$$\delta_0 \approx ka \left(\frac{\tanh \varkappa a}{\varkappa a} - 1 \right) \quad \text{linear in } ka$$

this is similar to a solid sphere behavior, $\delta_0 = -ka_0$ with an adjusted size

$$a_0 = -a \left(\frac{\tanh \varkappa a}{\varkappa a} - 1 \right)$$

a_0 is called <u>scattering length</u>

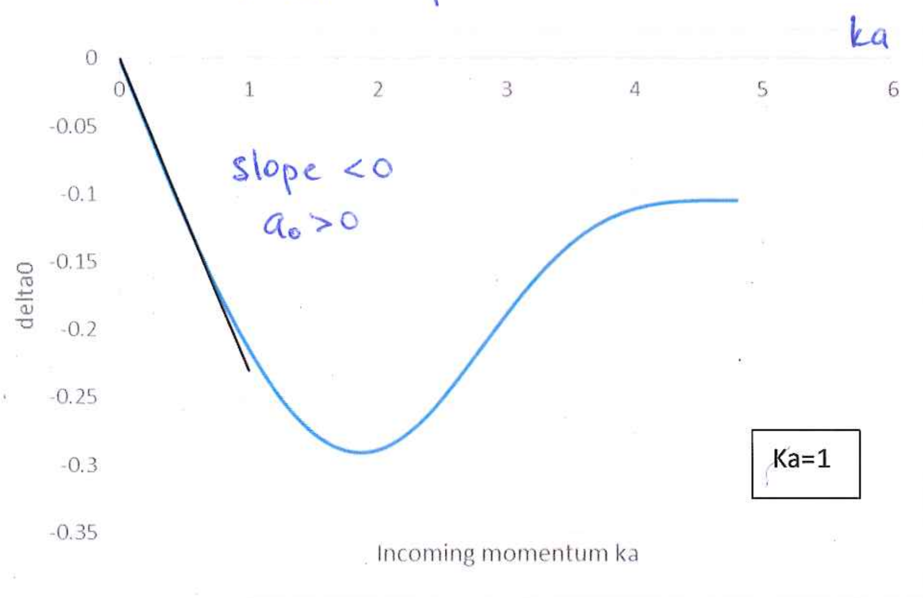
In general, one can define the scattering length for low-energy scattering

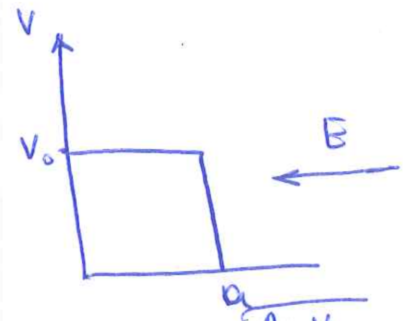
$$a_0 = -\lim_{k\to 0} \frac{d\delta_0}{dk}$$

Positive scattering length $\leftrightarrow$ repulsive potential

Attractive potential can have both positive and negative scattering length, depending on the strength of the potential

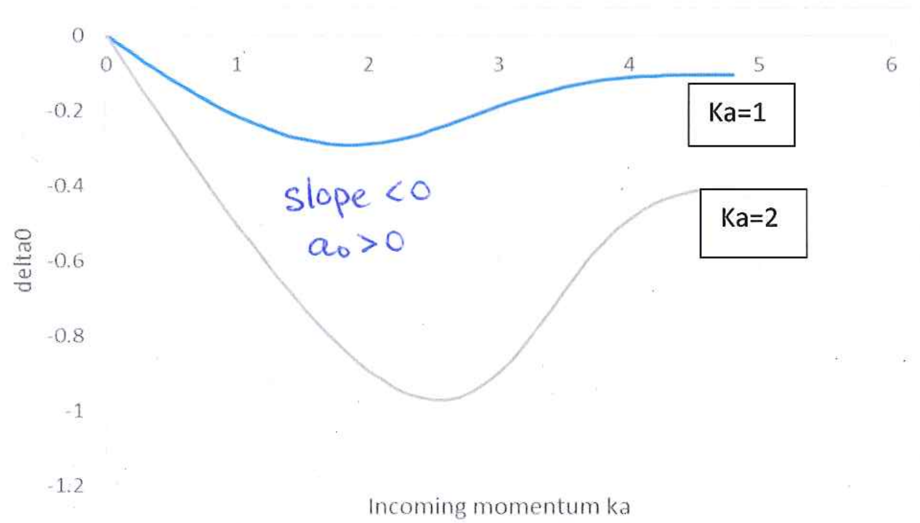
Repulsive potential





$$K = \varkappa = \sqrt{\frac{2mV_0}{\hbar^2}}$$

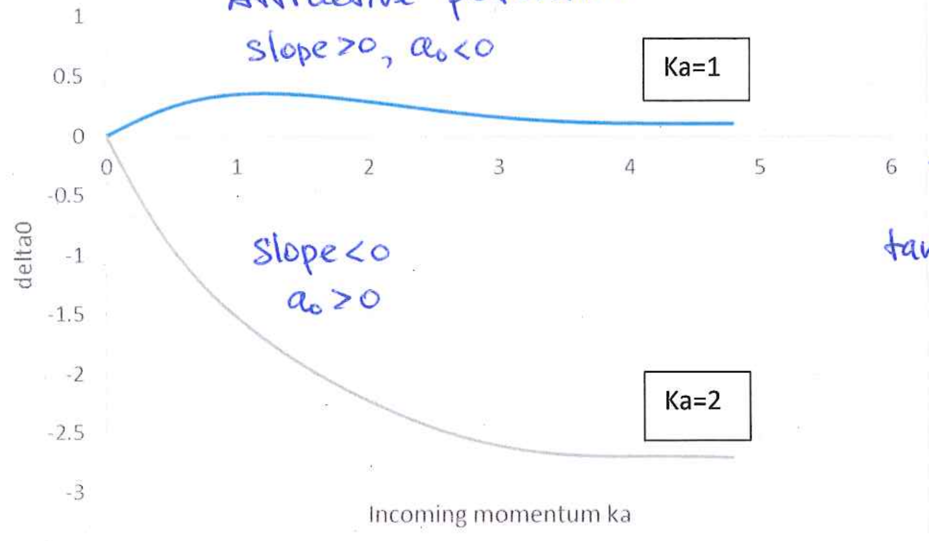
$$k = \sqrt{2mE/\hbar^2}$$

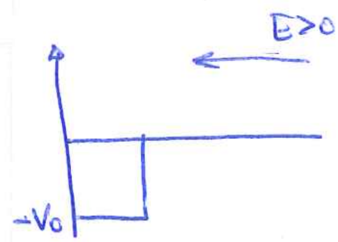


$$a_0 = -\lim_{k\to 0} \frac{d\delta_0}{dk}$$

a_0 is positive

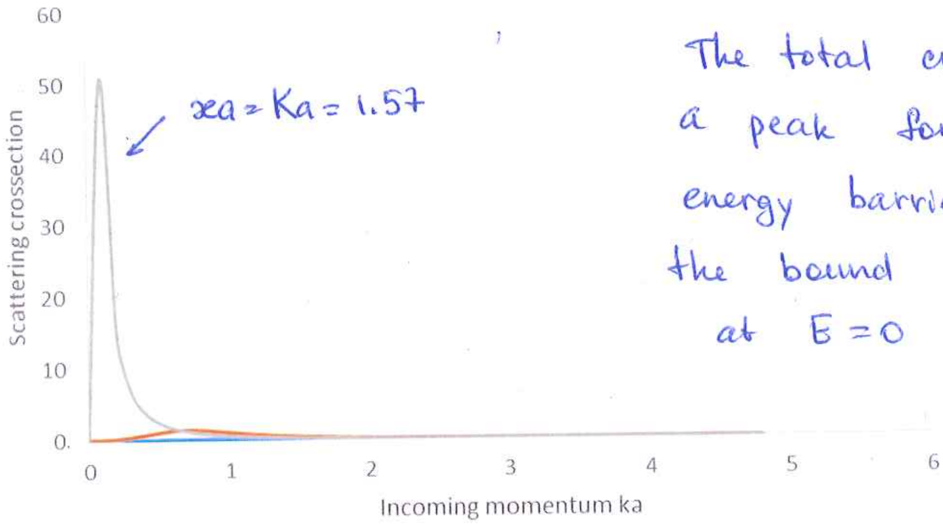
Attractive potential



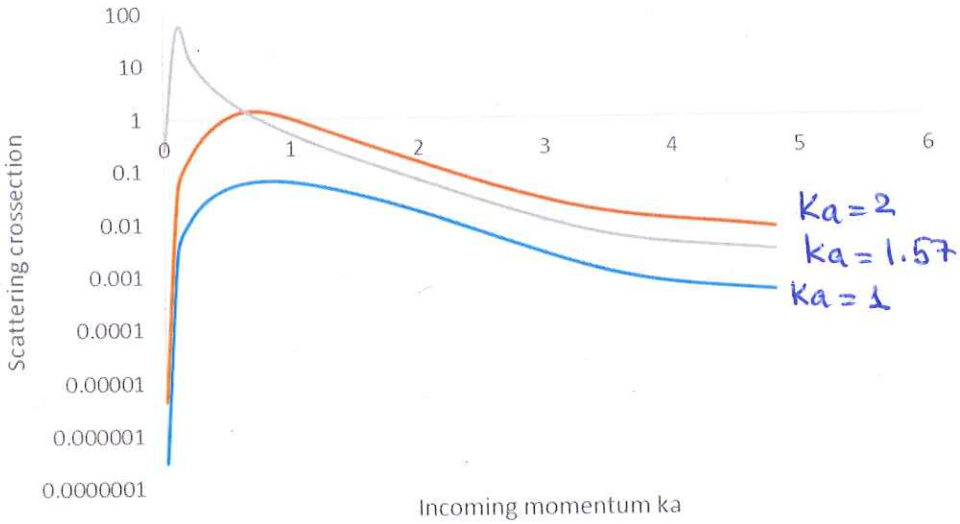


$$\tan\delta_0 = \frac{k}{\sqrt{k^2+\varkappa^2}} \tan\left[\sqrt{k^2+\varkappa^2}\,a\right] - ka$$

Resonant scattering



The total cross-section has a peak for values of energy barrier V_0 for which the bound state appears at $E=0$



$\varkappa a = Ka = 1.57$

$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0$$

$Ka = 2$

$ka = 1.57$

$Ka = 1$

Resonance scattering