Selection rules

$$\wp_{ab} = \langle \psi_a | \vec{d}\cdot\vec{e}_0 | \psi_b \rangle = \langle \psi_a | -er \overbrace{\vec{n}\cdot\vec{e}_0}^{\text{angular dependence}} | \psi_b \rangle \qquad \vec{n} = \frac{\vec{r}}{r}$$

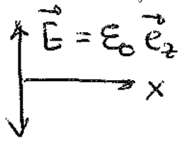
Atoms $\qquad \psi_{a,b} \to \psi_{n_{a,b}, l_{a,b}, m_{a,b}}$

$$\psi_{nlm} = A_{nlm} R_{nl}(r) \underbrace{L_l^m(\cos\theta)\, e^{im\varphi}}_{\text{angular dependence}}$$

$$\wp_{ab} = |A_{nlm}|^2 \underbrace{\langle R_{n_a l_a}(r) | -er | R_{n_b l_b}(r) \rangle}_{\substack{\text{radial part, does not} \\ \text{contribute into} \\ \text{selection rules}}} \times \underbrace{\langle \psi_{l_a m_a}(\theta,\varphi) | \vec{n}\cdot\vec{e}_0 | \psi_{l_b m_b}(\theta,\varphi) \rangle}_{\substack{\text{angular part} \\ \text{provides selection rules} \\ \text{for } l \text{ and } m \\ \text{depending on light} \\ \text{polarization}}}$$

Linear polarization

a) $\vec{E} = \mathcal{E}_0 \vec{e}_z$

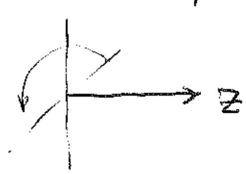


$\vec{e}_0 = \vec{e}_z \qquad (\vec{n}\cdot\vec{e}_0) = \cos\theta \qquad \sim L_1^0(\cos\theta)$

$$\langle \text{angular part} \rangle = \underbrace{\int (L_{l_a}^{m_a}(\cos\theta))^* L_{l_b}^{m_b}(\cos\theta) \cdot \cos\theta \, d\cos\theta}_{\sim \delta_{|l_a - l_b|,1}} \times \underbrace{\int e^{-im_a\varphi} e^{im_b\varphi} \, d\varphi}_{\delta_{m_a m_b}}$$

z - polarized light $\longrightarrow$ $|l_a - l_b| = 1$, $m_a = m_b$

b) Circularly polarized light



$\vec{e}_0 = \vec{e}_x \pm i\vec{e}_y$

$$\vec{n}\cdot\vec{e}_0 = \sin\theta\cos\varphi \pm i \sin\theta \sin\varphi = \sin\theta(\cos\varphi \pm i\sin\varphi) = \sin\theta\, e^{\pm i\varphi}$$

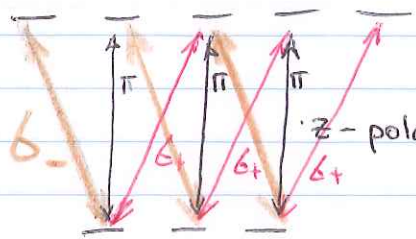
$\sim L_1^1(\cos\theta)$

$$\langle \text{ang. part} \rangle = \underbrace{\int (L_{l_a}^{m_a}(\cos\theta))^* L_{l_b}^{m_b}(\cos\theta) \sin\theta \, d\cos\theta}_{\sim \delta_{|l_a - l_b|,1}} \underbrace{\int e^{-im_a\varphi} e^{\pm i\varphi} e^{im_b\varphi} \, d\varphi}_{\delta_{m_b, m_a \pm 1}}$$

σ_+ - polarization $(\vec{e}_x + i\vec{e}_y)$: $|l_a - l_b| = 1$, $m_b = m_a + 1$

σ_- - polarization $(\vec{e}_x - i\vec{e}_y)$: $|l_a - l_b| = 1$, $m_b = m_a - 1$

$\ell_b = \ell_a \pm 1$
m_b



z-polarized light

ℓ_a, m_a -2 -1 0 1 2

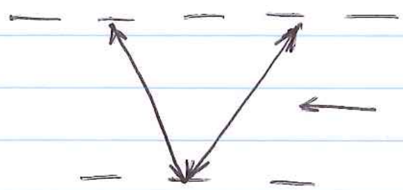
What if we have x-polarized light?

$\vec{n}\cdot\vec{e}_0 = \sin\theta\cos\varphi = \frac{1}{2}\sin\theta\,(e^{i\varphi} + e^{-i\varphi})$

Same integral for $|\ell_a - \ell_b| = 1$

φ- integral

$$\frac{1}{2}\int e^{-im_a\varphi}\,(e^{i\varphi} + e^{-i\varphi})\,e^{im_b\varphi}\,d\varphi = \frac{1}{2}\,\delta(m_a - 1, m_b) + \frac{1}{2}\,\delta(m_a + 1, m_b)$$



both transitions are enabled simultaneously (equal strength)