PHYS 314
Problem set # 6 *(due March 25)*
Each problem is 10 points, unless stated otherwise.

Griffiths: 5.16, 5.18

Q1 In class we have shown that two identical particles can either be found closer or farther from each other, depending on their statistics. In this problem we will be able to calculate the average "exchange force" that is responsible.

Consider a pair of free identical particles of mass m. For simplicity, suppose that they are moving in one dimension and neglect their spin variables. Each particle is described in terms of a real wave function, well-localized around points $+a$ and $-a$ respectively. For definiteness, assume that $\psi_\pm(x) = (\beta/\pi)^{1/4} exp\{-\frac{\beta}{2}(x \pm a)^2\}$.
(a) Calculate the total energy of the two particles assuming that they are well-localized (i.e. by assuming that $\beta \gg 1/a^2$, and their wave function overlap is negligible).
(b) Assuming that the two particles are *fermions*, write down the wave function of the system and calculate the expectation value of the energy. Show that in this case there is an *effective repulsion* between them. *Hint: One can find the effective force by evaluating the change in energy of the system, resulting from a variation in the distance of the two particles, $F = -d\langle E\rangle/d(2a)$.*
(c) Compare with the case of two identical bosons.

Q2 Consider N identical particles. Assume that their interactions can be neglected and that the Hamiltonian of the system is the sum of N identical one-particle Hamiltonians with known eigenvalues E_i:
$\hat{H} = \sum_{a=1}^{N} \hat{H}_a,\ \hat{H}_a|i\rangle_a = E_i|i\rangle_a$
(a) What is the energy of the ground state if these particles are spin-0 bosons? What if they are spin-1/2 fermions?
(b) Consider the case of three such particles and write down the corresponding ground-state wave functions.

Q3 Three identical spin-1 bosots are in the same orbital state, described by the wavefunction $\phi(\vec{r})$. Write down all possible normalized spin wavefunctions of the system. How many distinct states the system have? What are possible values of the total spin?