PHYS 314
Problem set # 3 *(due February 11)*
Each problem is 10 points, unless stated otherwise.

Griffiths, Ch. 6: 6.11, 6.19 (5 points each), 6.39

Q1 In class we are discussing linear Zeeman effect. However, there is also a quadratic Zeeman effect, given by the following Hamiltonian:
$\hat{H}_Q = \frac{1}{8}\alpha^2 B^2(x^2+y^2)$,
where B is the magnetic field in z-direction, and α is the fine structure constant.
Calculate the first order corrections $V_n^{(1)}$ to the energies of the hydrogen S-states (i.e. states with $\ell = 0$) due to the quadratic Zeeman effect. You may find this expression useful:
$\langle n\,\ell\,s\,j\,m_j|r^2|n\,\ell\,s\,j\,m_j\rangle = n^2\left[5n^2+1-3\ell(\ell+1)\right]/2$.

Q2 Calculate the first-order correction to the ground and first excited states of a one dimensional harmonic oscillator due to the relativistic correction to its kinetic energy. The mass of the oscillator is m, and its natural frequency is ω.
What would be an analog of "fine structure constant" in this system?

Q3 Let's explore the Zeeman effect for the hyperfine structure. We can describe the total perturbation Hamiltonian for hyperfine interaction and Zeeman effect for the ground state of a hydrogen atom as:
$\hat{H}' = a\hat{\vec{I}}\cdot\hat{\vec{S}}/\hbar^2 + k_1\hat{S}_z/\hbar - k_2\hat{I}_z/\hbar$,
where $\vec{S}$ is the spin of an electron in $\ell = 0$ state, $\vec{I}$ is the spin of a proton, and $k_{1,2}$ are the positive constants ($k_1 = g_s\mu_B B$ and $k_2 = g_p\mu_n B$, $k_1 \gg k_2$). Clearly, this state is now four-fold degenerate, since there are two possible orientations for both electron and proton spins. In further calculations, it may be convenient to use the following basis states: $\uparrow_e\uparrow_p$ (e.g., $m_S = +1/2, m_I = 1/2$), $\downarrow_e\downarrow_p$, $\uparrow_e\downarrow_p$ and $\downarrow_e\uparrow_p$. In the absence of the perturbation $\hat{H}'$ all four levels are degenerate. The ultimate goal of this problem is to find the first-order corrections to the energies of these four states without making any assumptions about the strength of the magnetic field.
Step one: By writing the first term of $\hat{H}'$ as $\frac{a}{2\hbar^2}(\hat{I}_+\hat{S}_- + \hat{I}_-\hat{S}_+) + \frac{a}{\hbar^2}\hat{I}_z\hat{S}_z$, show that the matrix form of the perturbation hamiltonian $\hat{H}'$ can be written as:

$$\hat{H}' = \begin{pmatrix} \frac{a}{4}+\frac{k_1-k_2}{2} & 0 & 0 & 0 \\ 0 & \frac{a}{4}-\frac{k_1-k_2}{2} & 0 & 0 \\ 0 & 0 & -\frac{a}{4}+\frac{k_1+k_2}{2} & \frac{a}{2} \\ 0 & 0 & \frac{a}{2} & -\frac{a}{4}-\frac{k_1+k_2}{2} \end{pmatrix}$$

Step two: Calculate the four energy corrections using $det|\hat{H}'-\lambda| = 0$.
Show that your solution is consistent with the expectations for the weak and strong magnetic field regimes.