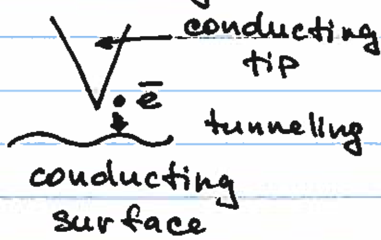
# Quantum tunneling

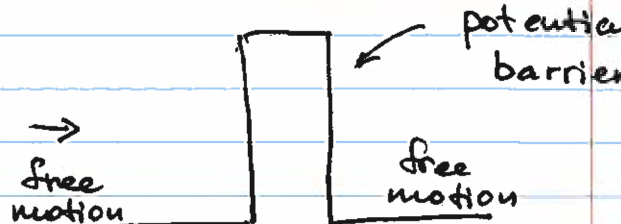
Tunneling: the possibility for a quantum particle to penetrate a classically forbidden region between two classically allowed region

Nuclear fusion $H^+ + H^+ \rightarrow He^{2+} + \ldots$

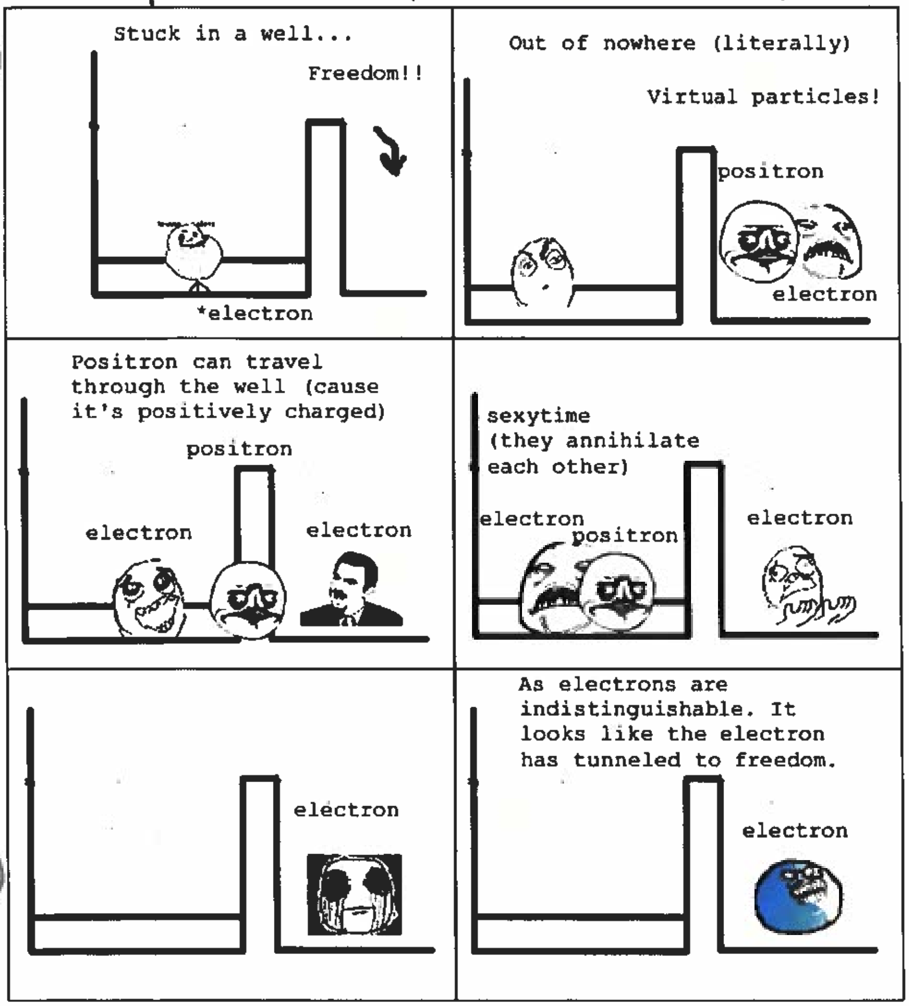

Protons must have an ability to penetrate Coulumn barrier to collocate and fuse

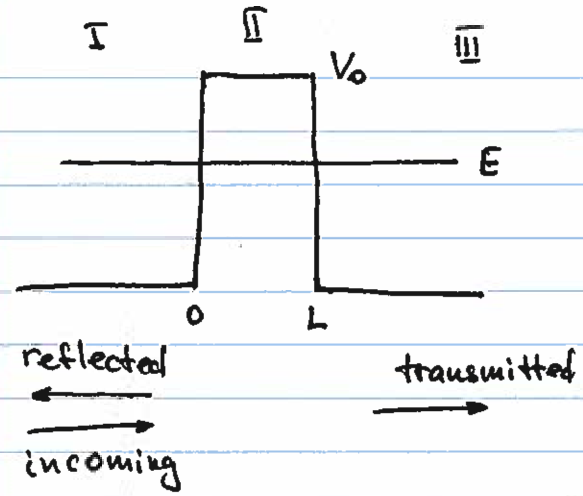
Scanning electron microscope



possible explanation of quantum tunneling






$$\psi(x) = \begin{cases} A e^{ikx} + B e^{-ikx} & x<0 \\ D e^{-qx} + F e^{qx} & 0<x<L \\ C e^{ik(x-L)} & x>L \end{cases}$$

Classically allowed region:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$$

$$k = \sqrt{2mE/\hbar^2}$$

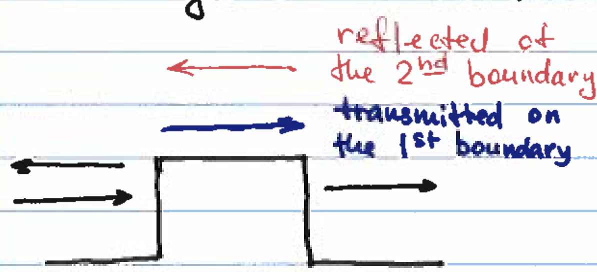
Classically forbidden region:

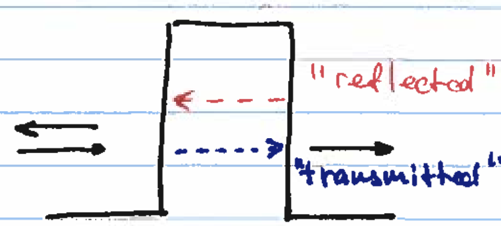
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = \frac{2m(V_0-E)}{\hbar^2}\psi = q^2\psi$$

$$q = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

~~Three~~ Intuitive ways to think about wave function in the classically forbidden region: ~~the~~ vs. classically allowed region




in case of evanescent waves they don't really travel, but it helps to "visualize" the wavefunction structure

Boundary conditions

$x = 0$: $\quad \psi_I(0) = \psi_{II}(0) \qquad A + B = D + F$

$\qquad \psi_I'(0) = \psi_{II}'(0) \qquad ikA - ikB = -qD + qF$

$x = L$: $\quad \psi_{II}(L) = \psi_{III}(L) \qquad De^{-qL} + Fe^{qL} = C$

$\qquad \psi_{II}'(L) = \psi_{III}'(L) \qquad -qDe^{-qL} + Fqe^{qL} = ikC$

solving these ~~for~~ four equations, (pg 230-231 of Townsen textbook)

$$T = \left|\frac{C}{A}\right|^2 = \frac{1}{1 + \left(\frac{k^2+q^2}{2kq}\right)^2 \sinh^2 qL} \overset{qL \gg 1}{\approx} \left(\frac{4kq}{k^2+q^2}\right)^2 e^{-2qL}$$

$$\sinh qL = \frac{1}{2}\left(e^{qL} + e^{-qL}\right) \xrightarrow{qL \gg 1} \frac{1}{2} e^{qL}$$

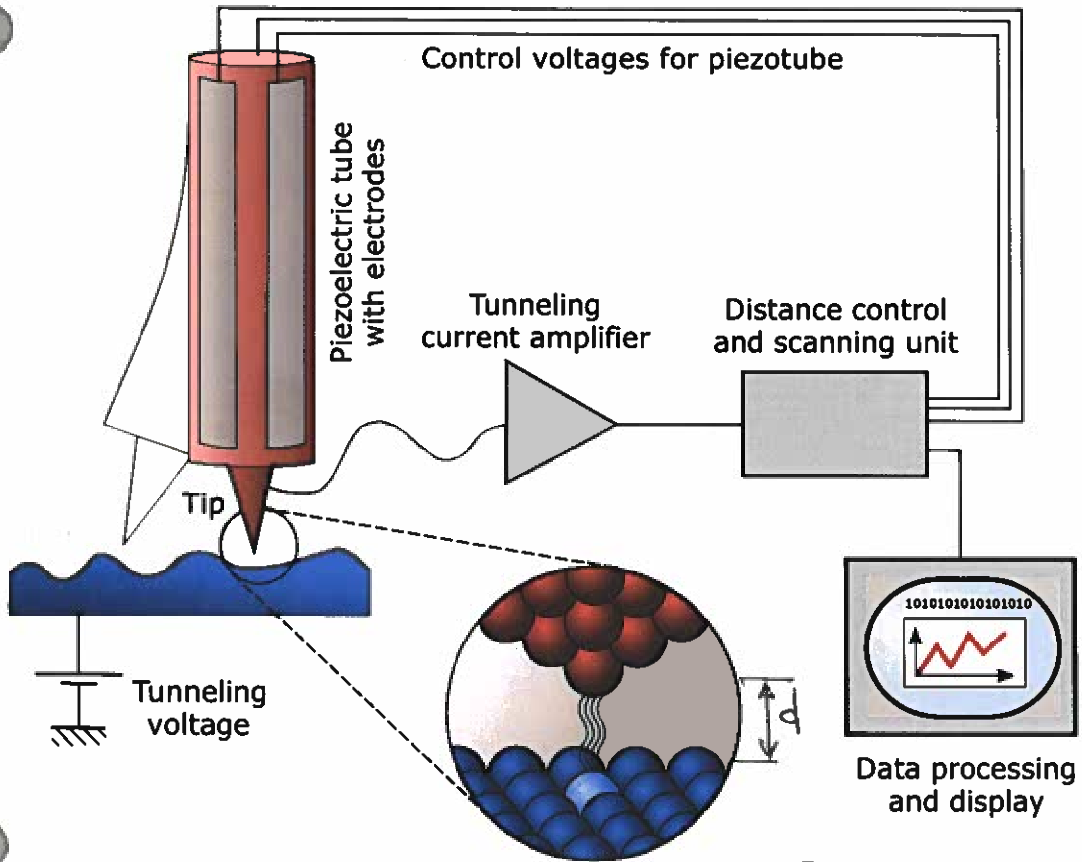
$$\frac{1}{1 + \left(\frac{k^2+q^2}{2kq}\right)^2 \sinh^2 qL} \longrightarrow \frac{1}{1 + \left(\frac{k^2+q^2}{4kq}\right)^2 e^{qL}} \approx \left(\frac{4kq}{k^2+q^2}\right)^2 e^{-qL}$$

The transmission through a "thick" barrier ($qL \gg 1$) has exponential dependence on the barrier thickness.

$$q \sim k \sim \frac{2\pi}{\lambda} \qquad e^{-qL} \sim e^{-2\pi L/\lambda}$$

~~ste~~ Measuring ~~tunneling~~ tunneling probability

~~allowed~~ ~~to~~ allows to measure distances comparable to particle de Broigle wavelength ($\sim 0.1 - 1\,nm$)




Tunneling current $I \propto e^{-d/\lambda}$

$\lambda$ - electron's wavelength

Extreme sensitivity to tiny distance variations