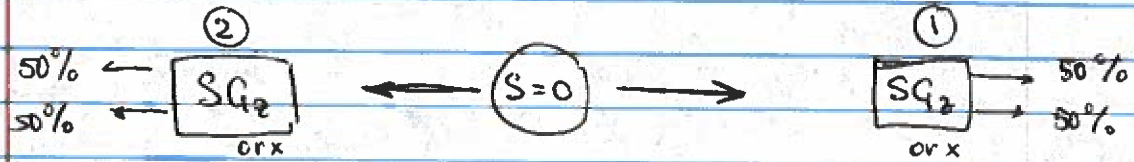
Quantum entanglement

$S=0$ state of two spin $1/2$ particles

$|0,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ in z-basis

$= \frac{1}{\sqrt{2}}(|+x,-x\rangle - |-x,+x\rangle)$

in short, two spins are always anti-correlated



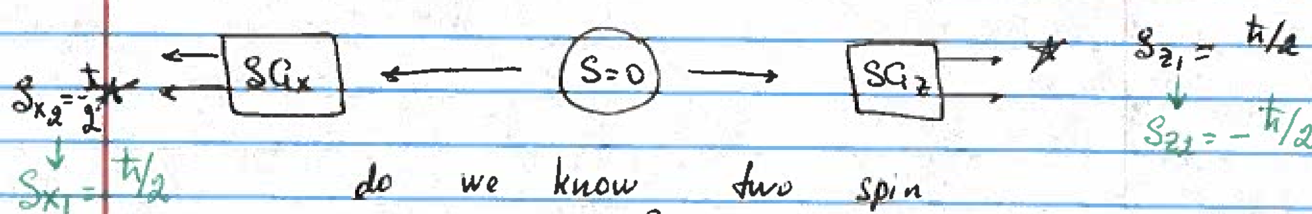
if ① +z then ② −z and vice versa

if ① +x then ② −x ——— " ———

always

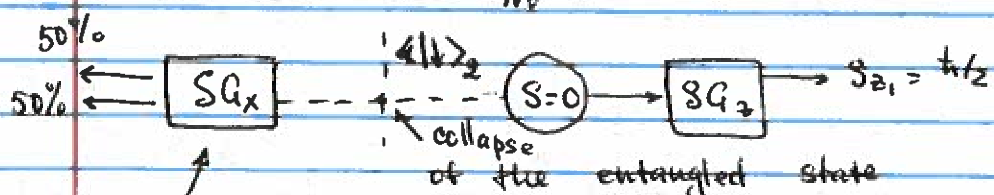
It appear that the selection of measurement type in ① affect the outcome of ②

Is it a way to overcome the uncertainty principle



do we know two spin components ?

No



this measurement adds nothing to the knowedge of the particle 1.

EPR paradox → quantum mechanics appears non-local.
(measurement in one location instantaneously affect the outcome of the other! at any separation)

Possible solution — hidden variable theory

$\{S_{2z}, S_{2x}\}$ ← $(S=0)$ → $\{S_z, S_x\}$

$\{-z, -x\}$ $\qquad\qquad\qquad$ $\{+z, +x\}$

Because of the total momentum $S=0$ the two spin components of each particles are always opposite

①	②
$\{+z, +x\}$	$\{-z, -x\}$
$\{+z, -x\}$	$\{-z, +x\}$
$\{-z, +x\}$	$\{+z, -x\}$
$\{-z, -x\}$	$\{+z, +x\}$

If $+z$ is measured for ①, then ② is either $\{-z, -x\}$ or $\{-z, +x\}$
So if we measure its z-component, we will always have $-z$, but if we measure S_x → 50/50 for both orentation

So either theory describes the physical reality, and we cannot test it?

Bell's inequality

If we assume that the hidden-variable theory is valid, then for three possible measurement orientations two particles must be in 8 possible states

	①	②
1:	$\{+a, +b, +c\}$	$\{-a, -b, -c\}$
2:	$\{+a, +b, -c\}$	$\{-a, -b, +c\}$
3:	$\{+a, -b, +c\}$	$\{-a, +b, -c\}$
4:	$\{+a, +b, -c\}$	$\{-a, +b, +c\}$
5:	$\{-a, +b, +c\}$	$\{+a, -b, +c\}$
6:	$\{-a, +b, -c\}$	$\{+a, -b, +c\}$
7:	$\{-a, -b, +c\}$	$\{+a, +b, -c\}$
8:	$\{-a, -b, -c\}$	$\{+a, +b, +c\}$

$$P(+a,+b) = P_3 + P_4$$
$$P(+a,+c) = P_2 + P_4$$
$$P(+c,+b) = P_3 + P_7$$

$$P(+a,+b) = P_3 + P_4 \leq P(+a,+c) + P(+c,+b) =$$
$$= P_3 + P_4 + \underbrace{P_2 + P_7}_{\geq 0}$$

If we can identify the conditions for which $P(+a,+b) > P(+a,+c) + P(+c,+b)$ we would have proven that HVT is not valid.

Let's look at QM treatment

$|-n\rangle \longleftarrow (S=0) \longrightarrow$ $\boxed{SG_{\vec{n}}}$

$|+n\rangle = \cos\frac{\theta}{2}|+z\rangle + \sin\frac{\theta}{2}|-z\rangle$ with 50% chance

$|-n\rangle = \sin\frac{\theta}{2}|+z\rangle + \cos\frac{\theta}{2}|-z\rangle$

θ angle
$\varphi = 0$

$$P(\underset{a}{+\theta}, \underset{b}{+0}) = \frac{1}{2}\cdot\sin^2\frac{\theta}{2}$$

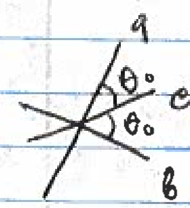
Since rotating the apparatus must not change anything

$$P(+a,+b) = \frac{1}{2}\sin^2\frac{\theta_{ab}}{2}$$

$$P(+a,+b) = \frac{1}{2}\sin^2\frac{\theta_{ab}}{2}$$

$$P(+a,+b) + P(+c,+b) = = \frac{1}{2}\sin^2\frac{\theta_{ac}}{2} + \frac{1}{2}\sin^2\frac{\theta_{bc}}{2}$$

If $\theta_{ac} = \theta_{bc} = \theta_0$ and $\theta_{ab} = 2\theta_0$



$\sin^2\theta_0$ vs $2\sin^2\theta_0/2$

$4\,\sin^2\theta/2\,\cos^2\theta/2$ vs $2\,\sin^2\theta/2$ (the $\sin^2\theta/2$ factors are struck out)

$\cos^2\theta/2$ vs $1/2$

for $\theta/2 < \pi/4$

Bell's inequality is violated

Quantum teleportation

Two-particle entangle-state basis — Bell states

$$|\Psi_{12}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right)$$

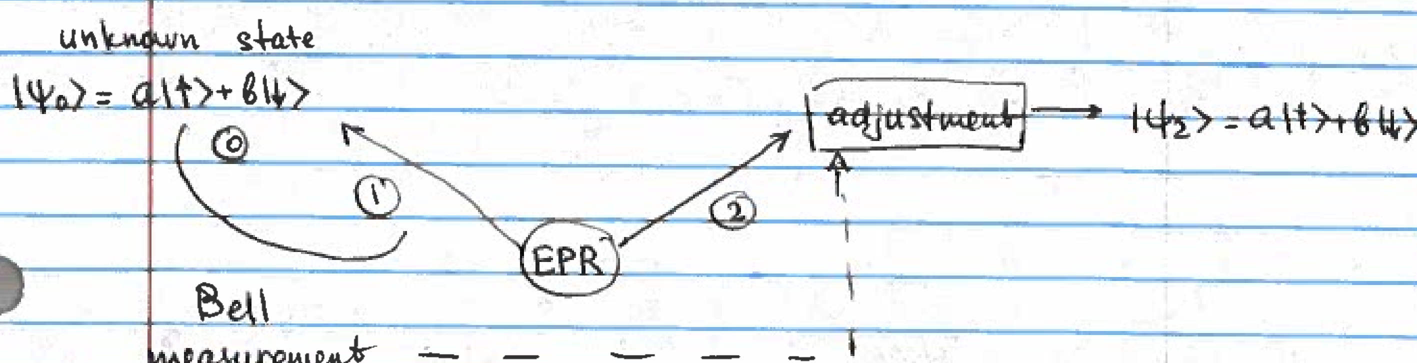
$$|\Phi_{12}^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right)$$

The general idea behind the teleportation:

1. We need to replicate a state of a single spin-1/2 particle in a different location.

We cannot measure its state and then recreate it, as a single measurement doesn't allow to gain full information about its state

However, we can make a joint two-photon state measurement with one half of the entangled pair, such that this measurement collapses the other half to the desired state.



EPR pair in $|\Psi_{12}^{-}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ state

Test particle

$|\psi_0\rangle = a|\uparrow\rangle + b|\downarrow\rangle$

A three particle state $|\psi_{012}\rangle = |\psi_0\rangle \otimes |\Psi_{12}^{-}\rangle =$

$= (a|\uparrow\rangle + b|\downarrow\rangle)\frac{1}{\sqrt{2}}(|\uparrow\ \rangle - |\downarrow\ \rangle) =$

$= \frac{a}{\sqrt{2}}|\uparrow\uparrow\downarrow\rangle - \frac{a}{\sqrt{2}}|\uparrow\downarrow\uparrow\rangle + \frac{b}{\sqrt{2}}|\downarrow\uparrow\downarrow\rangle - \frac{b}{\sqrt{2}}|\downarrow\downarrow\uparrow\rangle$

Next, we make a joint measurement with the test particle and the particle 1

$|\uparrow\uparrow\rangle = \frac{1}{\sqrt{2}}(\Phi_{01}^{+} + \Phi_{01}^{-})$ $\quad |\downarrow\downarrow\rangle = \frac{1}{\sqrt{2}}(\Phi_{01}^{+} - \Phi_{01}^{-})$

$|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}(\Psi_{01}^{+} + \Psi_{01}^{-})$ $\quad |\downarrow\uparrow\rangle = \frac{1}{\sqrt{2}}(\Psi_{01}^{+} - \Psi_{01}^{-})$

$= |\Phi_{01}^{+}\rangle\frac{1}{\sqrt{2}}\left(\frac{a}{\sqrt{2}}|\downarrow\rangle - \frac{b}{\sqrt{2}}|\uparrow\rangle\right) + |\Phi_{01}^{-}\rangle\frac{1}{2}(+a|\downarrow\rangle + b|\uparrow\rangle) +$

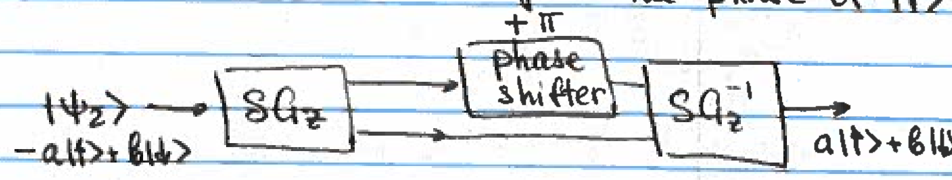
$+ |\Psi_{01}^{+}\rangle\frac{1}{2}(-a|\uparrow\rangle + b|\downarrow\rangle) + |\Psi_{01}^{-}\rangle\frac{1}{2}(-a|\uparrow\rangle - b|\downarrow\rangle)$

If we measure... then $|\psi_2\rangle =$

$|\Psi_{01}^{-}\rangle$ $\quad |\psi_2\rangle = a|\uparrow\rangle + b|\downarrow\rangle$ target

$|\Psi_{01}^{+}\rangle$ $\quad |\psi_2\rangle = -a|\uparrow\rangle + b|\downarrow\rangle$

need to change the phase of $|\uparrow\rangle$



if we measure then

$|\Phi_{01}^{-}\rangle$ $|\psi_2\rangle = a|\downarrow\rangle + b|\uparrow\rangle$

need to flip both spins

$|\psi_2\rangle$ ($b|\uparrow\rangle + a|\downarrow\rangle$) $\longrightarrow$ [NMR π-pulse] $\longrightarrow$ $b|\downarrow\rangle + a|\uparrow\rangle$ target

$|\Phi_{01}^{+}\rangle$ $|\psi_2\rangle = +a|\downarrow\rangle - b|\uparrow\rangle$

need to do both spin-flip and the phase adjustment

In any case, one can produce the copy of the original unknown state at the output

QM allows that since we never gain much direct information about a & b, so we did not destroy it in the process of measurements.