PHYS 313: Quantum Mechanics I

Problem set #8 (due November 20)

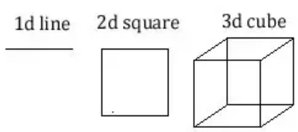
All problems are mandatory, unless marked otherwise. Each problem is 10 points.

Townsend, Ch. 9: 9.9, 9.11 (use information in Table 9.1 to find HCl moment of inertia), 9.19, 9.20(a,b) *Hint: this is a superposition problem, use the known expressions for $Y_{l,m}$ from (9.152a)*, 9.21

Q1: multidimentional quantum wells You are abducted by (very friendly) aliens who have access to words of different dimensions. You need to impress them with your knowledge of quantum mechanics by predicting the absorption spectrum of an infinite square quantum well in 1D, 2D and 3D of size a.

$$V(x) = \begin{cases} 0 & 0 < x_i \leq a \\ +\infty & otherwise \end{cases},$$

where x_i represent coordinates in one, two or three dimensions. (a) Find first 20 lowest eigenenergies of a free particle inside an infinite square well in the units of $\mathcal{E}_0 = \pi^2\hbar^2/(2ma^2)$. *I suggest you use some technology to make sure you don't miss any states, even MS Excel will be up to that task.*



(b) If you alien friends reveal they are in fact 11-dimensional beings, how many quantum numbers will you need to characterize a quantum state of an alien electron in an infinite 11D square well? What is the ground state and the first excited state energies of such system?

Q2 In this problem we explore the concept of degeneracy: the situation in more than one quantum state corresponds to the same eigenvalue of an operator. For example, consider a three-dimentional space, in which a certain set of orthonormal kets - $|1\rangle$, $|2\rangle$ and $|3\rangle$ - are used as the base kets. In this basis the two operators $\hat{A}$ and $\hat{B}$ can be expressed as (with a and b both real):

$$\hat{A} = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}, \; \hat{B} = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

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(a) It is easy to see that $\hat{A}$ has a degenerate spectrum, since $\hat{A}|1\rangle = a|1\rangle$, but $\hat{A}|2\rangle = -a|2\rangle$ and $\hat{A}|3\rangle = -a|3\rangle$, i.e., the states 2 and 3 yield the same eigenvalue. Does $\hat{B}$ also exhibits a degenerate spectrum?

(b) Show that $\hat{A}$ and $\hat{B}$ commute (and thus must have a set of common eigenstates).

(c) Find a new set of *orthonormal* kets which are simultaneously eigenkets of both $\hat{A}$ and $\hat{B}$. Specify the eigenvalues of $\hat{A}$ and $\hat{B}$ in each of the three eigenkets. Does your specification of the eigenvalues completely characterize each eigenket, or you have some freedom to choose how you define them?