PHYS 313: Quantum Mechanics I

Problem set # 5 *(due October 25)*

Mandatory problems (each problem is 10 points).

Townsend, Ch. 4: 4.4, 4.5, 4.11, 4.13 (*while this problem may look intimidating at first, it is actually very similar to ammonia maser problem we did in class*)
Townsend, Ch. 5: 5.5, 5.6

Q1 Neutrino oscillations
An elegant example of a quantum interference in a two-level system is neutrino oscillations (in reality, there are three flavors of neutrinos, but we'll limit ourselves with only two - electron neutrino ν_e and muon neutrino ν_μ. However, such defined "flavor" eigenstates do not match "mass eigenstates" m_1 and m_2.
Reminder: for a highly relativistic particles the total energy $E_{1,2}$ depends on the rest mass as
$E_{1,2} = \sqrt{p^2c^2 + m_{1,2}^2c^4} \approx pc\left(1 + \frac{m_{1,2}^2c^2}{2p^2}\right) \approx E_0 + \frac{m_{1,2}^2c^4}{2E_0}$, where $E_0 = pc$.
The relations between the flavor eigenstates $|\nu_e\rangle$ and $|\nu_m u\rangle$ and the mass eigenstates $|\nu_{1,2}\rangle$ are as follows:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle - \sin\theta|\nu_2\rangle \\ |\nu_\mu\rangle &= \sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \end{aligned}$$

Assuming that at $t = 0$ the electron neutrino left the sun, show that the probability to detect it as an electron neutrino at a distance $L = ct$ (where t is the travel time) is given by:

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\Delta m^2 c^4 \frac{L}{4E_0\hbar c}\right),$$

where $\Delta m^2 = m_1^2 - m_2^2$.

Optional problems
The problems below are not the part of the homework assignment, and should not be turned in for grading. However, you can use these problem if you want practice more to be better prepare for the second midterm. The solutions of the optional problems will be provided.
Townsend, Ch. 4: 4.6, 4.8, 4.12
Townsend, Ch. 5: 5.2, 5.4