PHYS 313: Quantum Mechanics I

Problem set # 3 *(due September 27)*

All problems are mandatory, unless marked otherwise. Each problem is 10 points.

In this and following homeworks Pauly matrices are defined as: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Townsend, Ch. 3: 3.5(a),3.9

Q1 Find the eigenvalues and eigenvectors of $\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
(You probably already know the answers, but nevertheless please derive them properly.)

Q2 One of the basic gates in quantum computing is a Hadamard gate, action of which is given by the following matrix $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.
(a) Find the output of the Hadamard gates if the input qubit is in $|\pm z\rangle$, $|\pm x\rangle$ or $|\pm y\rangle$ states.
(b) Find the eigenvectors and eigenvalues of H.

Q3 Let's do some quantum computing!
(a) Assuming that H is the Hadamard gate, and X, Y and Z are the Pauli gates, represented by the Pauli matrices $\hat{\sigma}_{x,y,z}$, show that $X = HZH$.
(b) Sometimes people use the notation of the $\sqrt{Z}$ gate to describe the $\pi/2$ phase gate $S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix}$. Confirm that this is not an unreasonable notation by proving that $S^2|\psi\rangle = Z|\psi\rangle$.