PHYS 313: Quantum Mechanics I

Problem set # 2 *(due September 20)*

All problems are mandatory, unless marked otherwise. Each problem is 10 points.

Townsend, Ch. 2: 2.4, 2.6, 2.7, 2.8, 2.9

Q1 This problem is mainly to train your matrix manipulation muscles. Using the expressions for states $|+\vec{n}\rangle$ and $|-\vec{n}\rangle$ in z basis (defined in 1.3 and 1.6, correspondingly), show that

$$|+\vec{n}\rangle\langle+\vec{n}| + |-\vec{n}\rangle\langle-\vec{n}| = \hat{1},$$

where $\hat{1}$ is an identity operator.

Q2 In class we have calculated the matrices for $\hat{J}_z$, $\hat{J}_z$ and $\hat{J}_z$ operators in z-basis:

$$\hat{J}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\ \hat{J}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\ \hat{J}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}.$$

(a) Very soon we will "discover" that these operators represent component of the angular momentum operator $\hat{\vec{J}}$, and will define the total angular momentum operator value as $\hat{J}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2$. Calculate the matrix representation of $\hat{J}^2$, using the matrices above. *(b)* We will also introduce raising and lowering operators $\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$. Even though you don't know yet why these operators are useful, you should be able to find their matrix representation. Please do so, and show that $\hat{J}_+^\dagger = \hat{J}_-$.