Intro to quantum mechanics - continued

In quantum mechanics we use a wave function (ψ or $|..\rangle$) to describe the state of the particle.
Measurement is separate act and often changes ψ.
Typically, a quantum state allows several possible outcome for a measurement

Eigenstates for a particular measurements
$\{|d_i\rangle\}$: $|d_1\rangle, |d_2\rangle \ldots |d_n\rangle$
If the ~~sta~~ system in the state $|d_i\rangle$ the outcome of the measurement is always d_i
In general, we will use the basis and decompose our actual state of the system into the combination of these states
$|\psi\rangle = c_1|d_1\rangle + c_2|d_2\rangle \ldots + c_n|d_n\rangle$
the propability of the outcome d_i is $|c_i|^2$

In many cases we use the measurement of energy to form the basis to describe the quantum state

Example: a free particle $E = P^2/2m = \frac{\hbar^2 k^2}{2m}$

$$k = \frac{2\pi}{\lambda} \qquad \lambda = \frac{2\pi\hbar}{p}$$

For a given value of kinetic energy $E = \frac{\hbar^2 k^2}{2m}$ the wave function is

$$\psi(x) = \frac{1}{\sqrt{2\pi}} e^{ikx - i\omega t} = \frac{1}{\sqrt{2\pi}} e^{i\frac{px}{\hbar} - i\frac{E}{\hbar}t}$$

particle completely delocolized plane wave

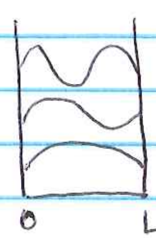
Bound states vs free particle

Classical waves: no boundaries – any frequency propagates
with boundaries – standing wave is formed, and only specific frequencies can be excited.

Frequency = energy in QM

Quantum wave functions: a particle can exist anywhere in space $\longrightarrow$ any energy is possible
If particle in classical world is localized in a certain region in space $\rightarrow$ energy spectrum is descrete, only specific values of energy is possible.

Examples: particle b/w to solid walls



Only the "standing" waves with $\psi = 0$ at the boundary are possible.

$L = \lambda/2, \lambda, 3\lambda/2 \ldots$

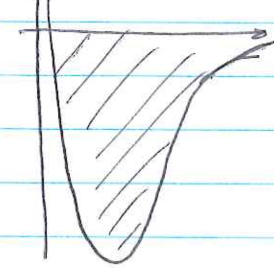
$$\lambda = 2L/n \qquad n = 1, 2, 3 \ldots$$

$$\frac{2\pi\hbar}{p} = \frac{2L}{n} \qquad p_n = \frac{\pi\hbar n}{L}$$

Eigen states, known energy
$$\begin{cases} E_n = \dfrac{p_n^2}{2m} = \dfrac{\pi^2\hbar^2 n^2}{2mL^2} & \text{energy spectrum} \\ \psi_n = \sqrt{\dfrac{2}{L}} \cos\left(\dfrac{\pi n x}{L}\right) = \dfrac{px}{\hbar} \end{cases}$$

In general if $E \neq E_n$
$$|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle + \ldots \qquad \text{(potentially infinite sum)}$$
$$E = |c_1|^2 E_1 + |c_2|^2 E_2 + \ldots$$

In an atom electrons are also bound: cannot move too far (Coulumb attraction of the nucleus) or too close (strong force)



For a bound state $E<0$, electrons are in limited region $\rightarrow$ descrite energy spectrum

Hydrogen atom

$$E_n = -\left(\frac{2\pi^2 m e^4}{\hbar^2}\right)\frac{1}{n^2} = -\frac{R_y}{n^2}$$

Rydberg constant