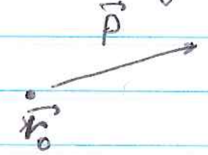
Crush course of quantum mechanics

Classical mechanics is fully determenistic, one can define the state of a system precisely, and predict with 100% certainty its future behavior.
For example, for a free particle we can predict its momentary position and momentum, and thus recreate its trajectory



$$v = \vec{P}/m$$
$$\vec{r}(t) = \vec{r}_0(t) + \vec{P}/m \cdot t$$
$$E = P^2/2m$$ kinetic energy

Quantum mechanics!
The state of a system is described using a wave function — probability amplitude. Using this function, one can predict the probability of particular measurement outcome (e.g., particle position or particle momentum). However, the act of measurement changes the system, so it is impossible to obtain complete information about the state of a system

Uncertanty principle
In quantum mechanics, the values of pairs of certain parameters cannot be measured at the same time with arbitrary precision
For example: position and momentum

$$\Delta x \cdot \Delta p \geq \hbar/2$$ $\left.\begin{matrix}\Delta x \\ \Delta p\end{matrix}\right\}$ uncertainty in position or momentum

Wave function — describes the state of the system ψ

Any measurement is described by an operator, acting on the wavefunction. This will predict the possible outcome of the measurement, but it will also change the state of the system

Example: a circularly-polarized photon and a beam splitter

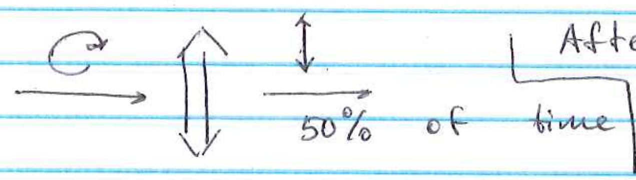
State of the system: $|\circlearrowright\rangle = (|\updownarrow\rangle + i\,|\leftrightarrow\rangle)\cdot\frac{1}{\sqrt{2}}$

Operator: polarization measurement

$$P_v\,|\updownarrow\rangle = |\updownarrow\rangle$$
$$P_v\,|\leftrightarrow\rangle = 0$$

Measurement: $P_v|\circlearrowright\rangle = \frac{1}{\sqrt{2}}\left(P_v|\updownarrow\rangle + i\,P_v|\leftrightarrow\rangle\right) =$

$$= \frac{1}{\sqrt{2}}|\updownarrow\rangle$$



After the polarizer the light polarization is linear

Procedure: we first decided what type of measurement we need to do, and found the "natural" — eigenstates — for this operator, then we wrote our current state in terms of these states.

Eiginstates of $\hat{A}$: $|\alpha_1\rangle, |\alpha_2\rangle \ldots |\alpha_n\rangle$

such that $\hat{A}\,|\alpha_1\rangle = \alpha_1\,|\alpha_1\rangle$

$\hat{A}\,|\alpha_2\rangle = \alpha_2\,|\alpha_2\rangle$

Measurement does not change the state, and the value of the $\hat{A}$ is well-defined.

If the system is not in one of these states, it can be presented in the $\{|\alpha\rangle\}$ basis

$$|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle + \dots\ c_n|\alpha_n\rangle$$

where $|c_i|^2 = P_i$ is a probability to find the ~~state~~ system in the state $|\alpha_i\rangle$

Since different states are associated ~~Example~~ with different values of $\hat{A}$, the state $|\psi\rangle$ does not have a well-defined value $\hat{A}$

Average value $\langle \hat{A} \rangle = |c_1|^2 \alpha_1 + |c_2|^2 \alpha_2 + \dots\ |c_n|^2 \alpha_2$

Momentum and position

Often we can write the wave-function in assuming that we can measure a position of the particle

$\psi(x)$ — function of position x

Probability ~~dent~~ density of detecting a particle at point~~s~~ ~~[x_0 {x_0+dx]~~ $p(x_0) = |\psi(x_0)|^2$

Probability of finding the particle in a certain range $[x_1, x_2]$

$$P(x_1, x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$$

Momentum operator $\hat{p} = -i\hbar \frac{\partial}{\partial x}$