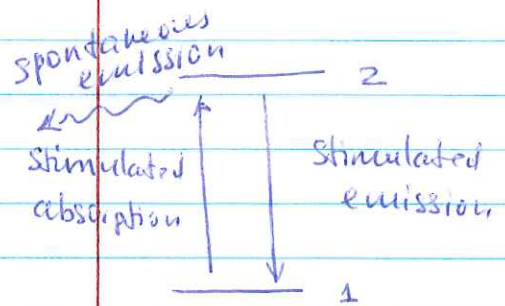
Lasers

Two-level atom



Spontaneous emission

$$\frac{dN_2}{dt} = -A_{21} N_2$$

Stimulated absorption

$$\frac{dN_2}{dt} = +B_{12} N_1 W(f)$$

Einstein coefficients

$$B_{21} = B_{12} \qquad = 1$$

$$A_{21} = \frac{4\pi f^3 n^3}{c^3} B_{21}$$

Stimulated emission

$$\frac{dN_2}{dt} = -B_{21} N_2 W(f)$$

Stimulated emission – adds photons $\propto N_2$

Stimulated absorption – remove photons $\propto N_1$

$$\frac{dI}{dz} = \left(\frac{dN_2}{dt}\Big|_{st} - \frac{dN_2}{dt}\Big|_{abs} \right) h\omega$$

$$\frac{dI}{dz} = B_{21} (N_2 - N_1) \frac{h\omega}{c} I(\omega) \underbrace{F(\omega)}_{\text{lineshape of the transition}}$$

e.g. $\propto \dfrac{1}{(\omega-\omega_0)^2 + \gamma^2/4}$

Gain coefficient for the e-m field $I \propto I_0 e^{g \cdot z}$

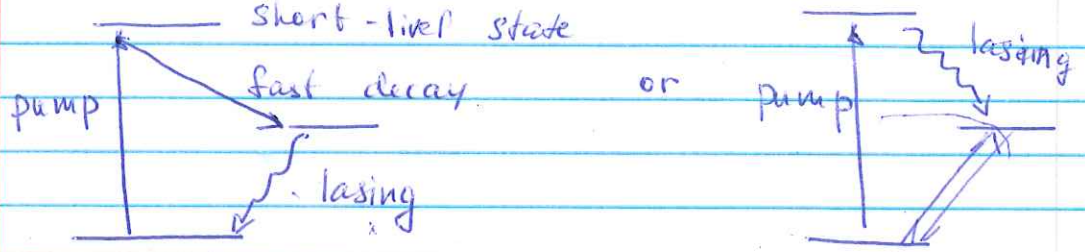
$$g(\omega) = A_{21} \left(\frac{c^2}{8\pi f^2} \right) F(\omega) \underbrace{(N_2 - N_1)}_{1} > 0 \quad \text{if } N_2 > N_1$$

population inversion

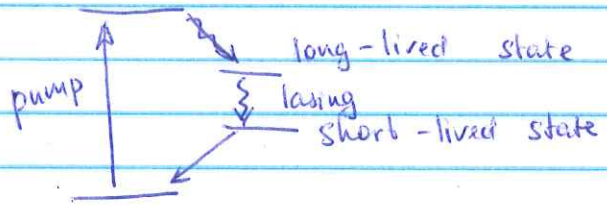
It is impossible to achieve stationary population inversion in a two-level system

Often, three- or four-level atoms (schemes) are used to achieve the population inversion.

Three-level scheme

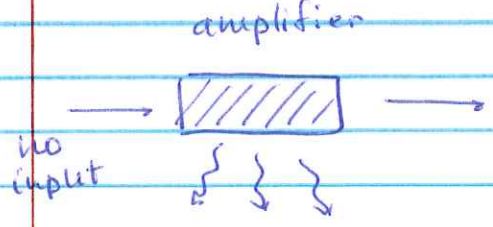


Four level system



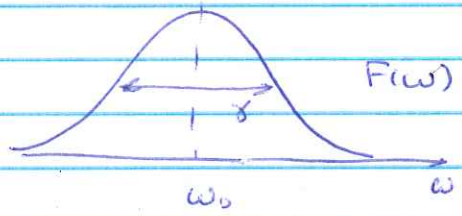
We now know how to amplify the transmitted light.
Laser does not need a "seed" optical fields.

Two main ingredients: population inversion (provides amplification) and optical cavity (feedback)



$$g(\omega) = \frac{c^2}{8\pi f^2 n^2} F(\omega) A_{21} \cdot (N_2 - N_1)$$

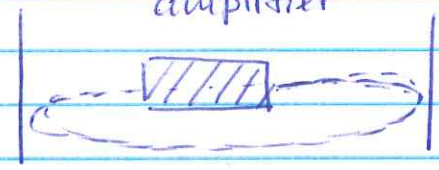
$F(\omega)$: lineshape of atomic transition



γ = $\frac{1}{\tau_{sp}^{(1)}} + \frac{1}{\tau_{sp}^{(2)}}$ lifetime of states

Since we often need a fast-decaying state involved, the gain spectral line is often very broad ⇒ amplified spontaneous emission is not very coherent

Need to artificially narrow the range of frequencies at which the gain may occure ⇒ optical cavity

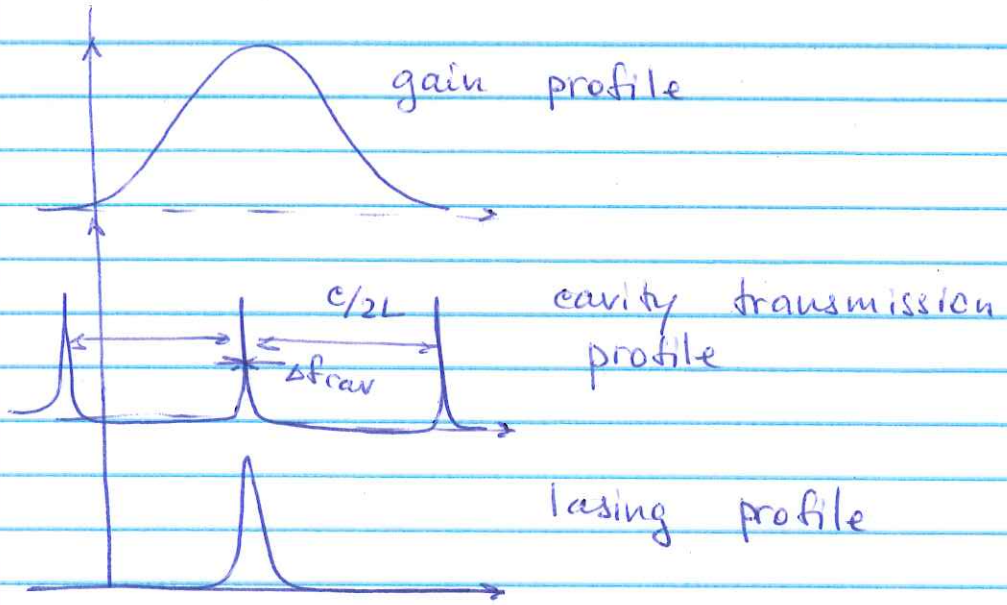


1. Light makes multiple roundtrips → effective length of an amplifying medium increases.

2. Only specific frequency can be transmitted (especially for a high-finesse-cavity)

$$f_{cavity} = \frac{c}{2L} \times m \qquad m = 1, 2, \ldots \text{ integer}$$

$$\Delta f_{cavity} = \frac{f_{cavity}}{F} \qquad F: \text{finesse}$$



Often, it is possible to control the exact frequency of cavity transmission, making the laser output tunable.