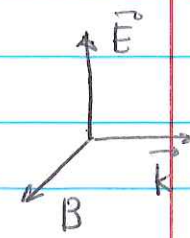
Electromagnetic wave

$$\vec{E} = \vec{E}_0 \cos(kz - \omega t + \varphi)$$
$$\vec{B} = \vec{B}_0 \cos(kz - \omega t + \varphi)$$



Electromagnetic wave is a transverse wave
$\vec{E} \perp \vec{B} \perp \vec{k}$ ($\vec{k}$ gives us propagation direction), $\vec{B}_0 = \frac{1}{c} \vec{E}_0$ (in vacuum)
$B_0 = \frac{1}{V} E_0$ (in a medium)

Maxwell's equations in a medium

$$\begin{cases} \nabla \cdot \vec{B} = 0 & \nabla \times \vec{E} = -\partial \vec{B}/\partial t \\ \nabla \cdot \vec{D} = 0 & \nabla \times \vec{H} = \partial \vec{D}/\partial t \end{cases}$$

here $\vec{D} = \underline{\underline{\varepsilon}} \varepsilon_0 \vec{E}$ $\quad \vec{B} = \mu \mu_0 \vec{H}$, but in many cases $\mu = 1$

ε – dialectric constant

Electro-magnetic wave carries energy

Energy density: $\frac{\text{energy}}{\text{unit volume}}$ $\quad u = \frac{1}{2}(\vec{D} \cdot \vec{E} + \vec{B} \cdot \vec{H})$.

$$u = \frac{1}{2}\left(\varepsilon \varepsilon_0 E^2 + \frac{1}{\mu \mu_0} B^2\right) = \frac{1}{2}\left(\varepsilon_0 \varepsilon E_0^2 \cos^2(kz - \omega t) + \frac{1}{\mu_0} B_0^2 \cos^2(kz - \omega t)\right)$$

instantaneous energy oscillates at frequency 2ω
What we could measure, is the density averaged over time $\gg 1/\omega$

$$\langle \cos^2(kz - \omega t) \rangle_T = 1/2$$

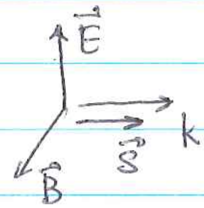
Average energy density

$$u = \frac{1}{4}\left(\varepsilon \varepsilon_0 E^2 + \frac{1}{\mu \mu_0} B^2\right) = \frac{1}{2} \varepsilon \varepsilon_0 E^2 \qquad B_0 = \frac{1}{V} E_0$$

How does the energy flow?

The energy flow rate, i.e the rate per unit area at which energy crosses a surface is given by Poynting vector

$$\vec{N} = \vec{E} \times \vec{H} = \frac{1}{\mu\mu_0} \vec{E} \times \vec{B}$$



$\vec{N}$ is in the same direction as $\vec{k}$

Phase front and energy travel in the same direction

$$|\vec{N}| = \frac{1}{\mu\mu_0} E_0 \cdot B_0 \cos^2(kz - \omega t)$$

$$\langle |\vec{N}| \rangle_t = \frac{1}{2} \frac{1}{\mu\mu_0} E_0 B_0 = \frac{1}{2} \frac{1}{\mu_0 \mu} \frac{1}{v} E_0^2 = \frac{1}{2} \sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}} E_0^2$$

(actually $\langle |\vec{N}| \rangle_t = v \cdot \langle u \rangle_t$)

I will assume that we always use time averaged values

In vacuum $\vec{N} = \frac{1}{2} E_0^2 \sqrt{\varepsilon_0 / \mu_0} = \frac{1}{2} \cancel{\varepsilon_0} E_0^2 / Z_0$

$Z_0 = \sqrt{\mu_0 / \varepsilon_0}$ impedance of free space

$\cancel{Z_0}$ · Impedance means generalized ~~resist~~ electrical resistance

$Z_0 = 377\ \Omega$

The magnitude of the pointing vector also gives intensity: energy per unit area per unit time (or power per unit area)

Momentum density of e-m wave
(momentum per unit volume, averaged in time)

$$\vec{p} = \frac{1}{c^2} \vec{N} = \frac{1}{2c} E_0^2 \cdot \vec{e}_z$$

How much momentum can radiation transfer?

$$\vec{P}_{total} = \vec{p} \cdot V = \vec{p} \cdot A \cdot c\Delta t$$

power

$$P_{total} = \frac{I}{c^2} \cdot A \cdot c \cdot \Delta t = \frac{I}{c} \underset{\uparrow \text{area}}{A} \cdot \Delta t = \frac{P}{c} \cdot \Delta t$$

$$\text{Radiation pressure} = \frac{\text{Force}}{\text{area}} = \frac{P_{total}/\Delta t}{A} = \frac{I/c \cdot \cancel{A}}{\cancel{A}} = \frac{I}{c}$$

On a sunny day with a perfect absorber

$$P_{radiation} \approx \frac{1300\ W/m^2}{3 \cdot 10^8\ m/s} \approx 4 \cdot 10^{-6}\ Pa$$

atmospheric pressure $\sim 10^5$ Pa

Atmospheric pressure is $\sim 2 \cdot 10^{10}$ times stronger than a radiation pressure