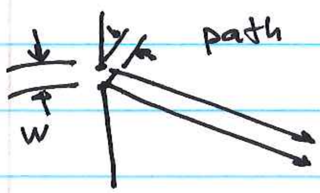
Diffraction

Interference : addition of light waves from two (or more) sources

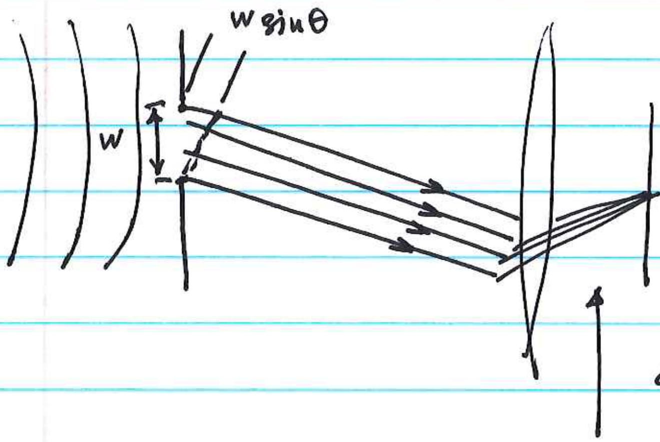
Diffraction: observed (usually) with a single source or obsticle of finite size.
Diffraction is also an interference effect

Two - slit interference – we previously assumed a very harrow slit ($w \ll \lambda$), so that any point within this slit oscillates in unison, no matter in what direction



path difference for the opposite end of the slits ~~one~~ is negligeable.

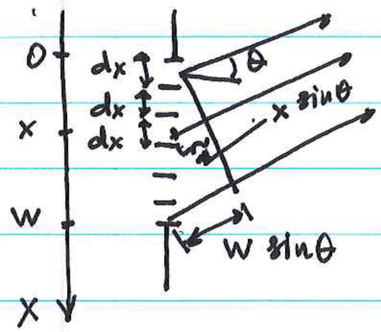
Wider slit



Depending on the location within a slit, each field is going to aquire different phase

all beams will interfere

One dimentional case



$$dE(x) = \frac{E_0}{w}\,dx\,\cos(kz-\omega t + kx\sin\theta)$$

$$E = \sum_{x=0}^{w} dE(x) = \int_0^w \frac{E_0}{w}\,dx\,\cos(kz-\omega t+\underline{kx}\sin\theta)$$

$$= \frac{E_0}{w}\int_0^w \cos(kx\sin\theta + kz-\omega t)\,dx =$$

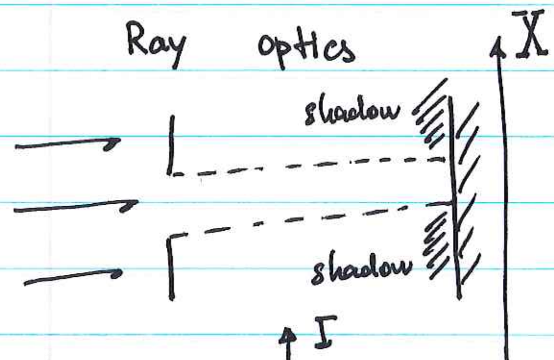
$$= \frac{E_0}{kw\sin\theta}\left[\sin(kw\sin\theta + kz-\omega t) - \sin(kz-\omega t)\right] =$$

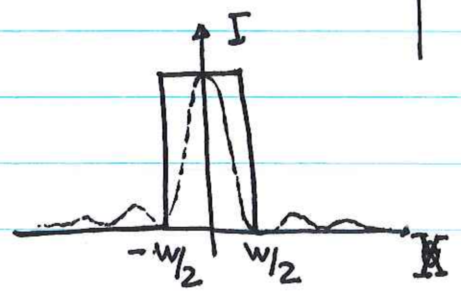
$$= \frac{2E_0}{kw\sin\theta}\sin\frac{kw\sin\theta}{2}\cos\left(kz-\omega t+\frac{kw\sin\theta}{2}\right)$$

$$I = \langle E^2\rangle_{time} = \frac{2E_0^2}{(kw\sin\theta)^2}\sin^2(kw\sin\theta/2) =$$

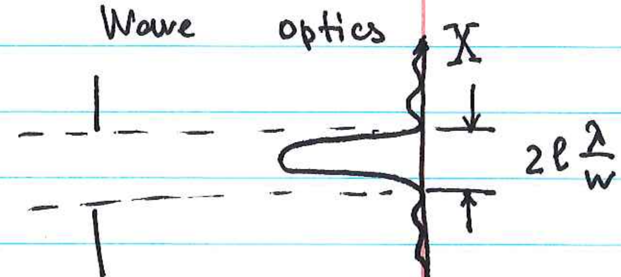
$$= \frac{E_0^2}{2}\left[\frac{\sin(kw\sin\theta/2)}{(kw\sin\theta/2)}\right]^2 = \frac{E_0^2}{2}\,\mathrm{sinc}^2\left(\frac{kw\sin\theta}{2}\right)$$

Ray optics





Wave optics



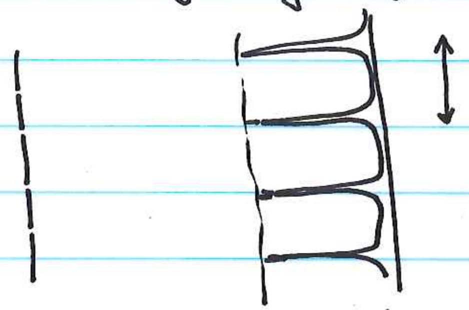
No sharp shadow!

$I=0$ $\quad \frac{kw\sin\theta}{2} = \pi = \frac{2\pi}{\lambda}\frac{w\sin\theta}{2}$

$\sin\theta = \frac{\lambda}{w}$

How would the finite slit size affect the interference pattern from a diffraction grating?

Ideal grating (tiny slits)

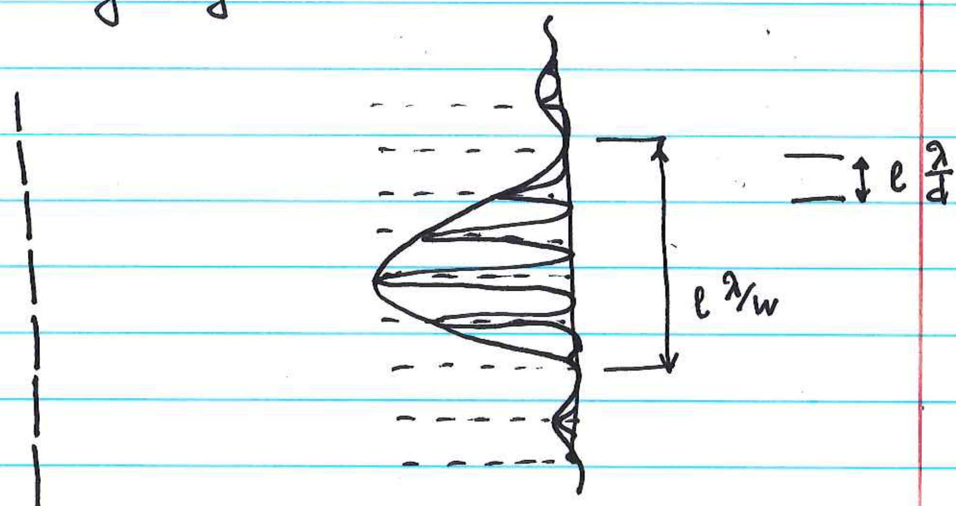


$\Delta x_{DG} = \ell \frac{\lambda}{d}$

$$I(\theta) = I_0 \left(\frac{\sin\left(\frac{\pi N d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)} \right)^2$$

same intensity

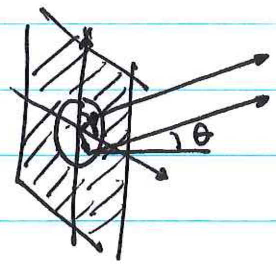
Real grating



$$I(\theta) = I_0 \left(\frac{\sin \frac{\pi w \sin\theta}{\lambda}}{\frac{\pi w \sin\theta}{\lambda}} \right)^2 \left(\frac{\sin\left(N \frac{\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)} \right)^2$$

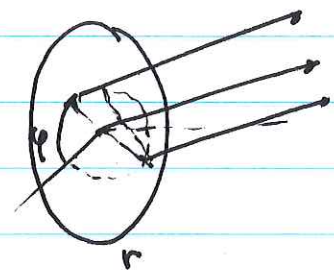
single slit diffraction — multiple slit interference

Diffraction on a circular hole



- Two-dimentional problem

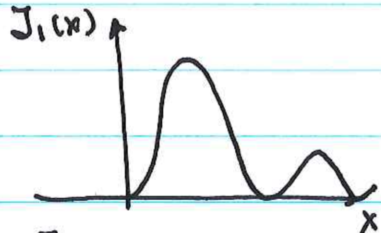
Phase shift depends both on θ and on the azimuthal angle

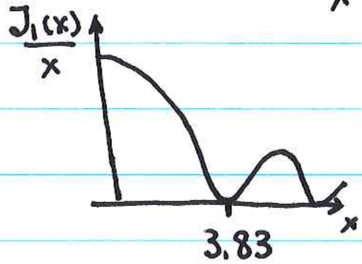


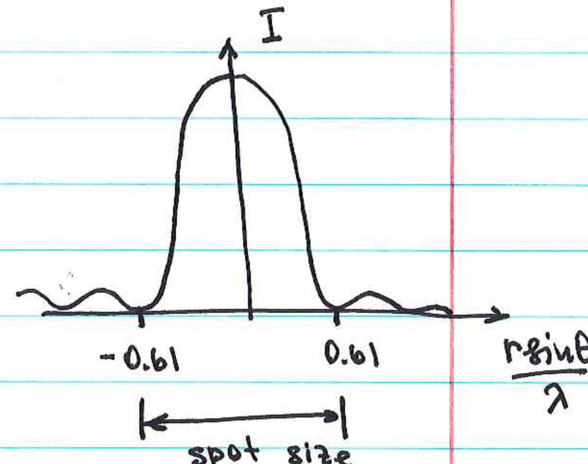
Two-dimentional integral

$$I(\theta) = I_0 \left[\frac{2 J_1(kr\sin\theta)}{(kr\sin\theta)} \right]^2$$

$J_1(x)$ - Bessel function (an ugly version of sine, often appears in 2D situations)







$$\sin\theta \approx 0.61 \frac{\lambda}{r} = 1.22 \frac{\lambda}{D}$$

Diffraction is limiting the resolution of an aperture. The larger is D, the smaller is the effect of the diffraction.

In general, diffraction prevents forming any sharped-edged shadows, instead on the borders ~~[illegible]~~ we will observe interference fringes