Problem set 4 (solutions)

Problem 1

Position 1: $d_o + d_i = L$ $\qquad d_i/d_o = -M_1$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$ Only L is known

Position 2: $d_i - a / d_o + a = -M_2 = -1/M_1$

$$\frac{1}{d_o+a} + \frac{1}{d_i-a} = \frac{1}{f}$$ a is also known

p1: $d_i = -M_1 d_o \qquad d_o - M_1 d_o = L \qquad d_o = \frac{L}{1-M_1}, \quad d_i = -\frac{M_1 L}{1-M_1}$

P2: $(-M_1)(d_i - a) = d_o + a$

$$\frac{M_1^2 L}{1-M_1} + aM_1 = \frac{L}{1-M_1} + a \quad \Rightarrow \quad M_1 = -\frac{a+L}{L-a} < 0$$ as expected

$$d_o = \frac{L}{1-M_1} = \frac{L-a}{2} \qquad d_i = L - d_o = \frac{L+a}{2}$$

$$f = \frac{d_o \cdot d_i}{d_o + d_i} = \frac{L^2 - a^2}{4L}$$

For $L = 135$ mm and $a = 45$m

$f = 30$mm

Problem 2

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

here $\underset{\text{(convex)}}{R_1 > 0}$ and $\underset{\text{(concave)}}{R_2 < 0}$

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{1.5m} - \frac{1}{2.0m}\right) = \frac{1}{12m}$$

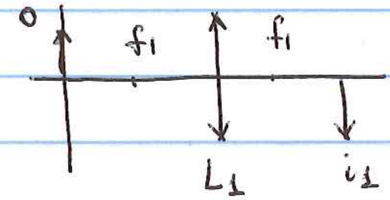
$$f = 12m$$

If the lens is reversed

we will have $\underset{\text{(concave)}}{R_1 < 0}$ and $\underset{\text{(convex)}}{R_2 > 0}$

so that f is the same

Problem 3



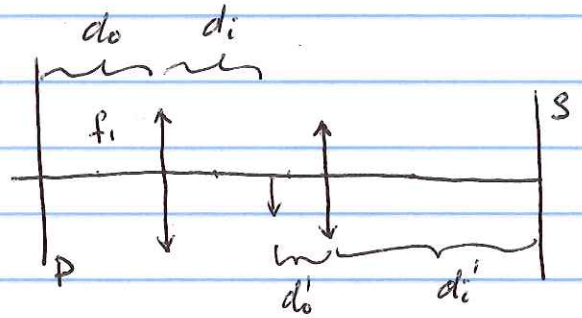
For the first lend $d_o = 2f_1$, thus the image will be formed at $d_i = 2f_1$ with magnification -1

Thus, magnification of the second lens is -3 and $d_i' = 3d_o'$

$$\frac{1}{d_o'} + \frac{1}{d_i'} = \frac{1}{d_o'} + \frac{1}{3d_o'} = \frac{1}{f_2}$$

$$d_o' = \frac{4}{3} f_2 = 26.7\,mm$$

$$d_i' = 4f_2 = 80\,mm$$



Distance b/w the object plane and the first lens $= 200\,mm$

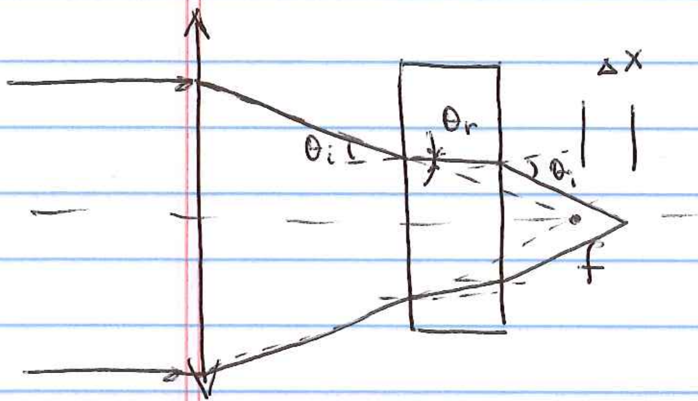
Distance b/w the two lenses $d_i + d_o' = 227m$

Distance from L_2 to the screen $= d_i' = 80\,mm$

Problem 4

Each portion of a lens divert the light beams from a particular point of the object to the particular point of an image. Covering some parts of a lens reduces the amount of light that forms the image, but preserves its shape.

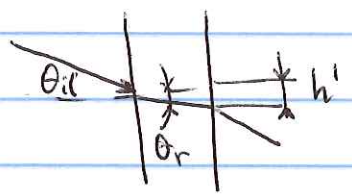
Problem 5



Inside the plate the beam travels at smaller angle $\sin\theta_i = n \sin\theta_r$

After the plate the angle is the same as before, but it takes larger distrance for the beams to converge at the focus

Vertical displacement inside the plate is $h' = d \tan\theta_r \approx d\theta_r$ in the paraxial approximation



Under the same approximation

$\theta_i = n\theta_r \Rightarrow \theta_r = \frac{1}{n}\theta_i$

Without the plate the beam descent by $h \approx d\theta_i$ at the same distance.

Thus, after the plate the beam must travel extra length to descend by $(h-h') = d\theta_i - d\frac{\theta_i}{n}$

This extra distance $\Delta x = \frac{h-h'}{\theta_i} = d(1-\frac{1}{n}) = d\left(\frac{n-1}{n}\right)$