# Wave-particle duality – for all!

We have discussed that ~~photons~~ light can behave either as a particle (photon) or a wave

| Wave description | Particle description |
|---|---|
| $f, \lambda$, ~~electric field~~ | momentum $p = h/\lambda$ |
| $\vec{E} = \vec{E}_0 \cos(\frac{2\pi}{\lambda} z - 2\pi f t + \varphi)$ | energy $E = hf$ |
| interference arises from coherent superposition of e-m field amplitudes | interference? |

→ to reconsile these two paradigm we have to describe the light using a quantum <u>wave</u> <u>function</u> (probability wave)

A single photon with defined wavelength $\lambda$ and ~~p~~ freq $f$

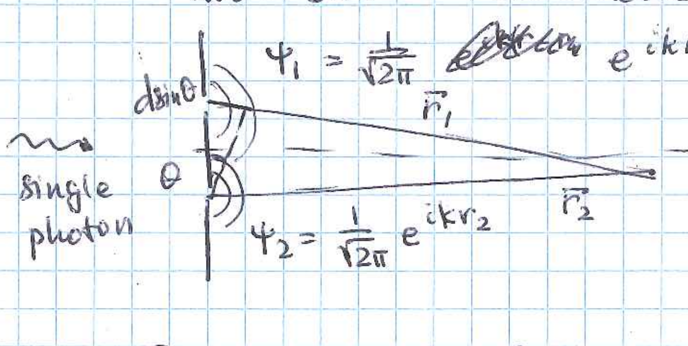
$$\psi(z,t) = \frac{1}{\sqrt{2\pi}} e^{ikz - i\omega t + i\varphi}$$

$k = \frac{2\pi}{\lambda}$ wave vector (spatial frequency)

$\omega = 2\pi f$ cyclic frequency

A probability of finding a photon at a particular outcome is determined by the absolute value squared of the wave function, so the measurable values are not sensitive to the wave function phase.
However, if the wave function has several components (i.e photon can be in several states at once), the interference principle works for the wave functions, and the resulting outcome does depend on the relative phases b/w different paths.

Two-slit interference




$\psi_1 = \frac{1}{\sqrt{2\pi}}$ ~~[illegible]~~ $e^{ikr_1}$ $\quad \vec{r}_1$

$\psi_2 = \frac{1}{\sqrt{2\pi}} e^{ikr_2}$ $\quad \vec{r}_2$

Since we don't know which ~~hole~~ slit the photon went through,

$$\psi_{tot} = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

(50% chance – slit 1 $\left|\frac{1}{\sqrt{2}}\right|^2$
— " — – slit 2 — " —)

Probability to find it anywhere at a screen

$$P = \frac{1}{2}\left|(\psi_1 + \psi_2)\right|^2 = \frac{1}{2}\left|e^{ikr}\right|^2 \left|1 + e^{ikd\sin\theta}\right|^2 = \cos^2\left[\frac{kd\sin\theta}{2}\right]$$

But what about other particles?

de Broglie idea – all particles have some wave properties! (pilot waves)

Particle's ~~...~~ kinetic energy $E \longrightarrow f = h/E$

Particle's momentum $p \longrightarrow \lambda_p = \dfrac{h}{p}$

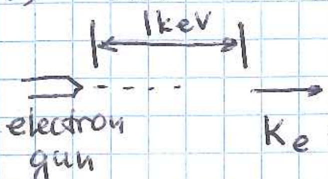
It is more common (and fun) to use $\hbar = h/2\pi$

$E = \hbar\omega$ $\quad \hbar = 10^{-34}\ J\cdot s$ de Broglie wavelength $\lambda_p = \dfrac{2\pi\hbar}{p} \Rightarrow k = p/\hbar$, $p = \hbar k$

Why we don't see wave nature manifistation for most particles? $\lambda_p$ is too small!

a) A Bowling ball $\quad p = 1kg \cdot 1\ m/s \Rightarrow \lambda \approx 6 \cdot 10^{-34}\ m$

b) "Slow" electrons




$K_e \approx 1keV$ (non-relativistic electrons)

$K_e \approx \dfrac{p_e^2}{2m} \quad p_e \approx \sqrt{2m_e K_e} \quad \lambda_e = \dfrac{2\pi\hbar}{p_e} \approx 0.04nm$

(comparable with X-rays)

One can see the diffraction if e-beam relects off ~~cristo~~ crystals.

c.) Proton/neutrons – 2000 time heavier at room temperature $\quad K \approx 0.03eV \quad (T = 300K)$

$\lambda_p = \dfrac{2\pi\hbar}{\sqrt{2m_p K}} \approx 0.16\ nm$ $\longrightarrow$ also can be used to probe ~~small~~ structure, including for bio/medical purposes!

Since electrons and protons are charged particles, they are often destractive for internal structures, but neutrons are very "gentle" (but one needs a reactor to produse them!)

Some helpful conversions for calculations

$hc = 1240\ eV\cdot nm$

$\lambda = \dfrac{h}{\sqrt{2mK}} = \dfrac{hc}{\sqrt{2(mc^2)K}}$

Electron: $K_e = 10^3 eV \quad mc^2 = 0.5\cdot MeV = 5\cdot 10^5 eV$

$\lambda_e = \dfrac{1240\ eV\cdot nm}{\sqrt{2\cdot 5\cdot 10^5 eV\cdot 10^3 eV}} = \dfrac{1250 eV\cdot nm}{\sqrt{10^9}eV} = \dfrac{1250\ eV\cdot nm}{3\cdot 10^4\ eV} \approx 0.04nm$

How can we make the de Broglie wavelength larger? We need to make ~~things~~ particles colder!

Ultracold atoms $T \approx 10\ nK \Rightarrow K = 3 \cdot 10^{-12}\,eV$

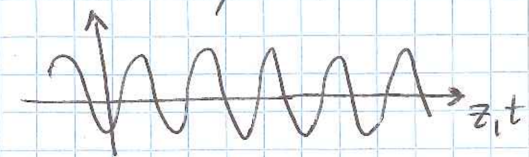
$$\lambda_p = \frac{1240\,eV \cdot nm}{\sqrt{2 \cdot 10^9\,eV \cdot 3 \cdot 10^{-12}\,eV}} = 1.6 \cdot 10^4\,nm = 16\,\mu m$$

observable scale!

Uncertainty principle

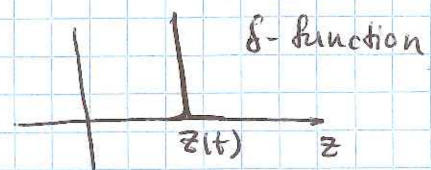
"Pure" wave — ~~can~~ exists uniformly in time and space

~~8e~~ $A\cos(kz - \omega t)$




complete delocalization
well-defined momentum

"Pure" particle — absolute localization
well-defined position




wave vs particle
momentum vs position

In classical physics we can know everything about any object, it is a completely deterministic description.

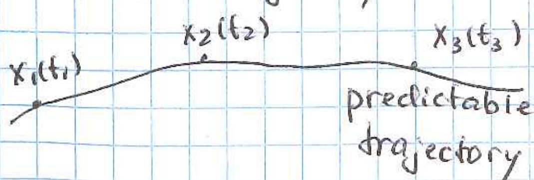
Since in quantum mechanics we are dealing with probabilistic ~~desc~~ description, we may not be able to receive the complete information

**Classical description**

deterministic

measurements don't affect the state of a system, so consequitive measurements can provide all information about all parameters

Classical trajectory

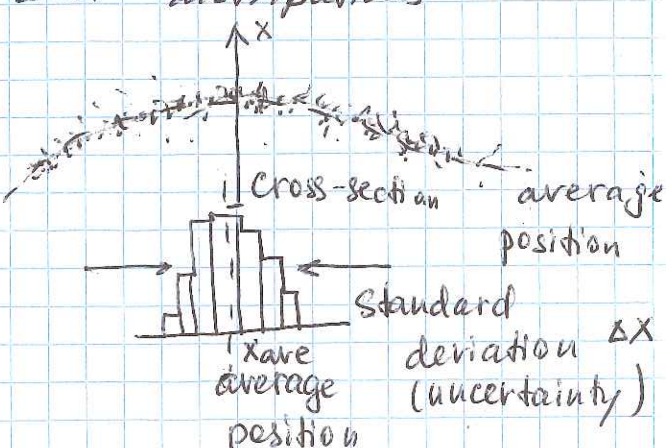



**Quantum description**

probabilistic:

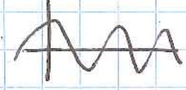
An act of measurement changes the state of a system, so only partial information can be obtained

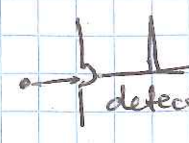
It is only possible to talk about <u>average</u> <u>values</u> and <u>distributions</u>



In quantum mechanics certain values cannot be measured simultaneously, i.e. a measurement of one parameter will change the outcome of a measurement ~~of~~ for another

Position and momentum of a particle are such two parameters

Two extremes: wave  $\vec{k}$ is precisely known ($\vec{p} = \hbar\vec{k}$) no position information

particle 

position is known but momentum information is lost

In general, it is possible to determine average values for position and momentum and their uncertainty

position: $x_{ave} \pm \Delta x$ momentum $p_{ave} \pm \Delta p$

Heisenberg uncertainty principle

$$\Delta x \cdot \Delta p \geq \hbar$$

Fundamental limitation on measurement precision

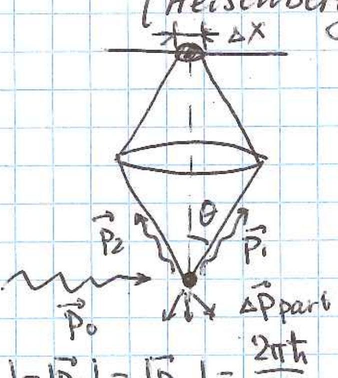
Another important uncertainty principles

Energy - time $\Delta E \cdot \Delta t \geq \hbar$ (energy conservation can be violated for a short time!)

Photon number - phase $\Delta n \cdot \Delta\varphi \geq 1$

(fundamental limit of optical measurements

Physical interpretation of the uncertainty principle (Heisenberg microscope)




$|\vec{p}_0| = |\vec{p}_1| = |\vec{p}_2| = \frac{2\pi\hbar}{\lambda}$

Limited optical resolution

$$\Delta x = \frac{\lambda}{2\sin\theta}$$

However, in order for a photon to be scattered into the cone of the lens' vision, its momentum must change direction, and due to the momentum conservation, a momentum kik is given to the particle

$$|\Delta\vec{p}_{photon}| = |-\Delta\vec{p}_{part}| \cong 2 p_0 \cdot \sin\theta = 2\frac{2\pi\hbar}{\lambda}\sin\theta$$

Thus, $\Delta x_{part} \cdot \Delta p_{part} \cong \frac{\cancel{\lambda}}{\cancel{2\sin\theta}} \cdot \frac{2\pi\hbar}{\cancel{\lambda}} \cdot \cancel{2\sin\theta} \sim 2\pi\hbar$

$$\Delta x_{part}\, \Delta p_{part} \sim 2\pi\hbar$$

correct order-of magnitude estimate

There is a rigorous ~~de~~ mathematical basis to