Relativity principle

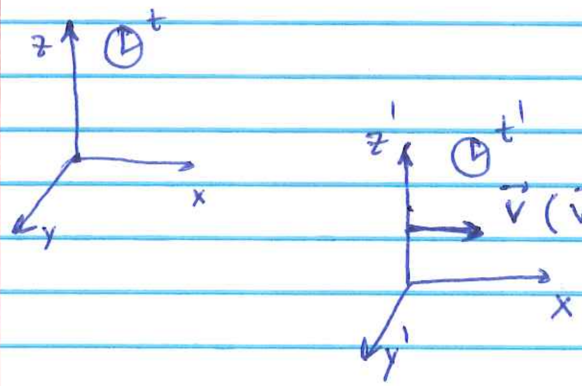
The laws of physics are the same in all inertial reference frames.

Reminder: inertial reference frames (RF) – RF moving without acceleration, i.e. moving with respect to each other with constant speed.

Two RF

object/event

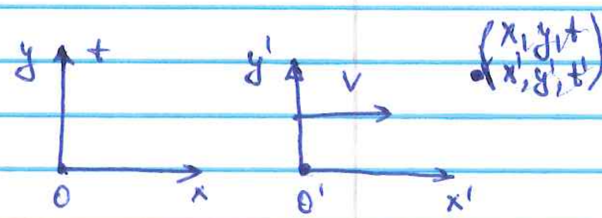
(x, y, z, t)
(x', y', z', t') } coordinates of the events in two RF



$\vec{v}$ ($\vec{v} = (v, 0, 0)$ – relative velocity)

Galelean transformation

$$\begin{cases} x' = x - vt \\ y' = y, \; z' = z \\ t' = t \quad \text{(universal time)} \end{cases}$$



Velocity addition

If an object moves with velocity $\vec{u} = (u_x, u_y, u_z)$

$$u_x = \frac{dx}{dt} \qquad u_x' = \frac{dx'}{dt'} = \frac{d(x - vt)}{dt} = u_x - V$$

$$\begin{cases} u_x' = u_x - V \\ u_y' = u_y \\ u_z' = u_z \end{cases}$$

Accelerations are the same in both RF $\left(a_x' = \frac{du_x'}{dt'} = a_x\right)$

Newtons laws are intact

Forces are invariant under the Galelean transformation, and do not change from one inertial reference frame to another.

However, there is a problem with Maxwell equations: their form changed under the Galelean transformation, and the speed of light was exactly c only in a particular selected reference frame.

Dilemma: Since Maxwell equations are universal law of physics, then the speed of light must be the same in all reference frames.

Einstein solution: time is not universal, and flows differently in different RFs.

Let's agree on some rules regarding how we measure and compare coordinates and times in different RF

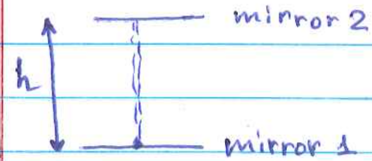
1. All clocks in the same RF are perfectly sinchronized and run the same way

2. Two observers, moving with respect to each other, always agree on their relative speed.

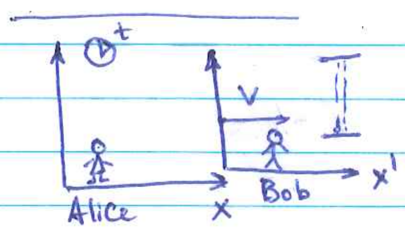
3. Two such observers in different RFs can <u>locally</u> synchronize their clocks using some physical event. So for example, we can make sure that at an instant $t' = t = 0$, $x' = x = 0$ (origin points are matched).

4. Distant events <u>cannot</u> be used

Ok, how we are going to build the clocks? Light clocks (since the speed of light is universal)

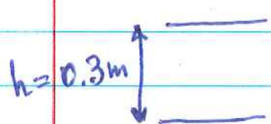


one bounce of light = 1 "tick"



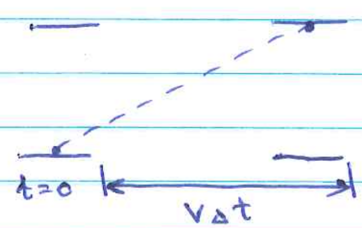
Let's install our light clock in Bob's RF, vertically, so that the distance b/w two mirrors is unaffected by the motion

Bob's RF: clocks are stationary



"tick'" $\underline{\Delta t'} = \frac{h}{c} = \frac{0.3\,m}{3\cdot 10^8\,m/s} = 1\,ns$

Alice's RF: mirrors are moving



The distance the light pulse has travelled

$\sqrt{h^2+(v\cdot\Delta t)^2} = c\cdot\Delta t$

universal speed of light

$$h^2+v^2\Delta t^2 = c^2\Delta t^2$$
$$(\Delta t')^2c^2+v^2\Delta t^2 = c^2\Delta t^2$$
$$(\Delta t)^2 = \frac{c^2(\Delta t')^2}{c^2-v^2} = \frac{(\Delta t')^2}{1-(v/c)^2}$$
$$\Delta t = \frac{\Delta t'}{\sqrt{1-(v/c)^2}} > \Delta t'$$

Time dilation

Lorentz factor $\gamma = \frac{1}{\sqrt{1-V^2/c^2}} \gtrsim 1$

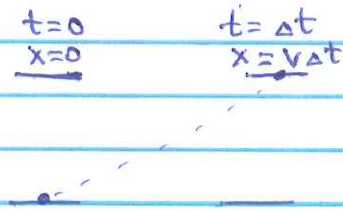
		$c = 3\cdot10^8$ m/s	Relativistic speed $\beta = V/c$	γ
	light	300000000 m/s		
human scale	car	30 m/s	10^{-7}	$1.000000000000005 = 1+5\cdot10^{-15}$
	missile	3000 m/s	10^{-5}	$1.00000000005 = 1+5\cdot10^{-11}$
	Sun escape velocity	600 000 m/s	$2\cdot10^{-3}$	$1+2\cdot10^{-6}$
			0.1	1.005
			0.5	1.155
			0.9	2.3
			0.99	7.1

For any "human" scale effects $(\gamma - 1)$ is usually too small to be measured.
That is why usually such relativistic effects are important only for accelerated particles / cosmic rays, etc.

Time dilation $\Delta t = \Delta t_0 \cdot \gamma$
Δt_0 – "proper time", measured in the reference frame where the clocks are stationary
In any other reference frame the time runs <u>faster</u>.

What about length?

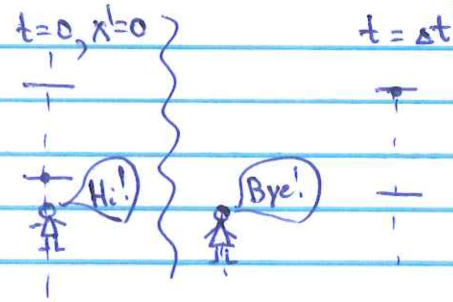
In Alice's RF



Distance Bob travelled

$\underbrace{L}_{\text{Proper length}} = v\Delta t$ (according to Alice)

In Bob's RF



Distance Bob travelled

$L' = v\Delta t'$ (according to Bob)

~~pro~~

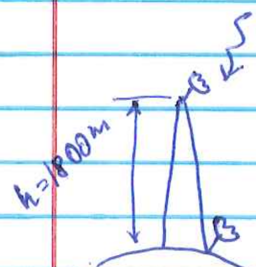
$L' = \frac{L}{\gamma} < L$

Length contraction: $L = v \cdot \Delta t = v\gamma\Delta t' = \gamma L'$

Bob will perceive ~~the~~ travelling a shorter ~~whereas~~ distance than Alice

~~In this case Alice~~

Cosmic rays measurements



μ-pions $v_\mu = 0.994c$ $\gamma = 9$

Particle lifetime: $\tau_0 = 1.5\,\mu s$

If at $t=0$ we have N_0 particles

after time $t = 1.5\,\mu s \rightarrow \frac{1}{2} N_0$

It takes $\Delta t = \frac{h}{c} = \frac{1800\,m}{3 \cdot 10^{8}\,m/s} = 6 \cdot 10^{-6}\,s = 6\,\mu s = 4\tau_0$

Top detector measures $N_0 = 570$ particles

~~We would expect~~ Expectation for the bottom detector

$N = \frac{1}{16} \cdot N_0 = 35$ events

Measurements: $N = 400$ events

Lifetime is measured in the pion RF
Hight of the mountain is measured in the Earth frame

Time dilation: in the Earth frame the pion lives longer! Lifetime $\tau_E = \gamma \tau_0 = 9 \cdot 1.5 \mu s = 13.5 \mu s$
So it takes only $\frac{1}{2}\tau_E$ to travel from top to bottom $\Rightarrow$ $N = 420$ particle (matches the experiment)

Length contraction: in the pion frame the mountain is shorter $h' = h/\gamma = 200 m$
travel time $\Delta t' = h'/c = 0.66 \mu s \approx \frac{1}{2}\tau_0$