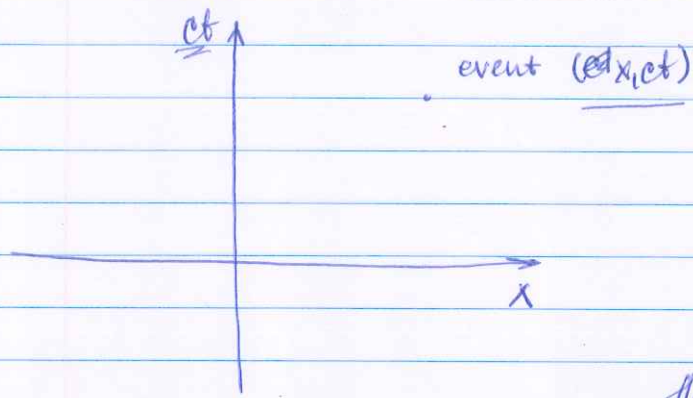
Space-time diagram
proposed by Minkowski in 1908

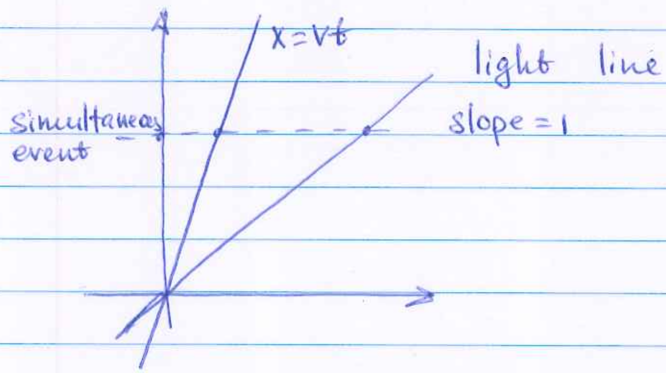
We restrict ourselves to 1D motion, so any events has two coordinates: x, t (or ct, to have same units)



x and ct are independent variables, describing where and when the event occure

Any curve/line in this space - wordline - describes the complete history and future of any object

$x=0$: all events at origin at any time
$t=0$: all events at any position at $t=0$



slope of the line

$$\text{slope} = c/v = \Delta(ct)/\Delta x = \frac{\Delta(ct)}{\Delta(vt)} > 1$$

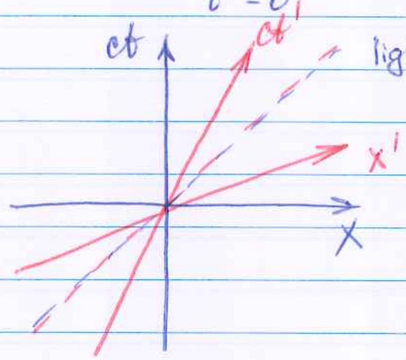
since speed of light is c in any reference frame, ~~the~~ the slope of the light line is always 1.

What about different reference frames?
Lorentz transformation

$$x' = \gamma(x - vt) = \gamma(x - v/c\,(ct))$$
$$ct' = \gamma(ct - v/c\,x)$$

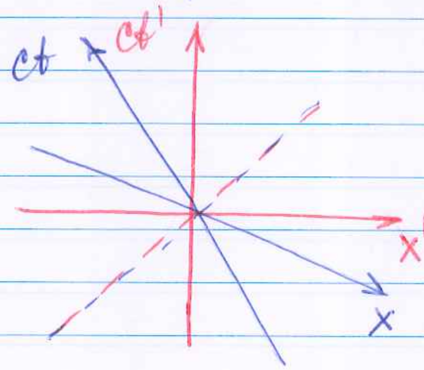
The ct' axis: points at origin in the moving RF ($x'=0$) at any time
$x'=0 \Rightarrow x = v/c \cdot ct$ (slope $= v/c$)

The x' axis: points at any position at $t'=0$
$t'=0$, $ct = v/c \cdot x \Rightarrow x = c/v \cdot (ct)$ (slope $= c/v$)



light line always have slope $=1$ in any RF

$\Updownarrow$ $v \leftrightarrow -v$



x-axis: $x' = -\frac{v}{c}(ct')$
t-axis: $x' = -\frac{c}{v}(ct')$

The Lorentz transformation is not a rotation, but rather stretch of the ~~coo~~ space-time coordinates. Thus, the length of a vector is no longer ~~an~~ same (Euclidian geometry does not work)

Relativistic invariant

$$\sqrt{(x_1-x_2)^2 - (ct_1 - ct_2)^2} =$$
$$= \sqrt{(x_1'-x_2')^2 - (ct_1' - ct_2')^2}$$

in all RFs