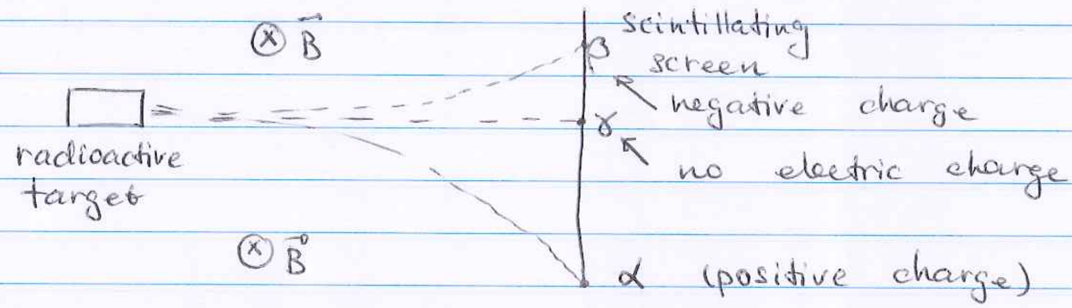
The birth of the particle physics

The structure of atoms were not known until very end of XIX century

Radioctivity (hottest physics topic at 1900s)



α-particles — He^{2+} (doubly-ionized He nuclei)
β-particles — electrons
γ — high-energy electro-magnetic radiation (photons)

Stable massive particles: proton (H^+) p $m_pc^2 = 938$ MeV
electron (e^-) $m_ec^2 = 0.51$ MeV

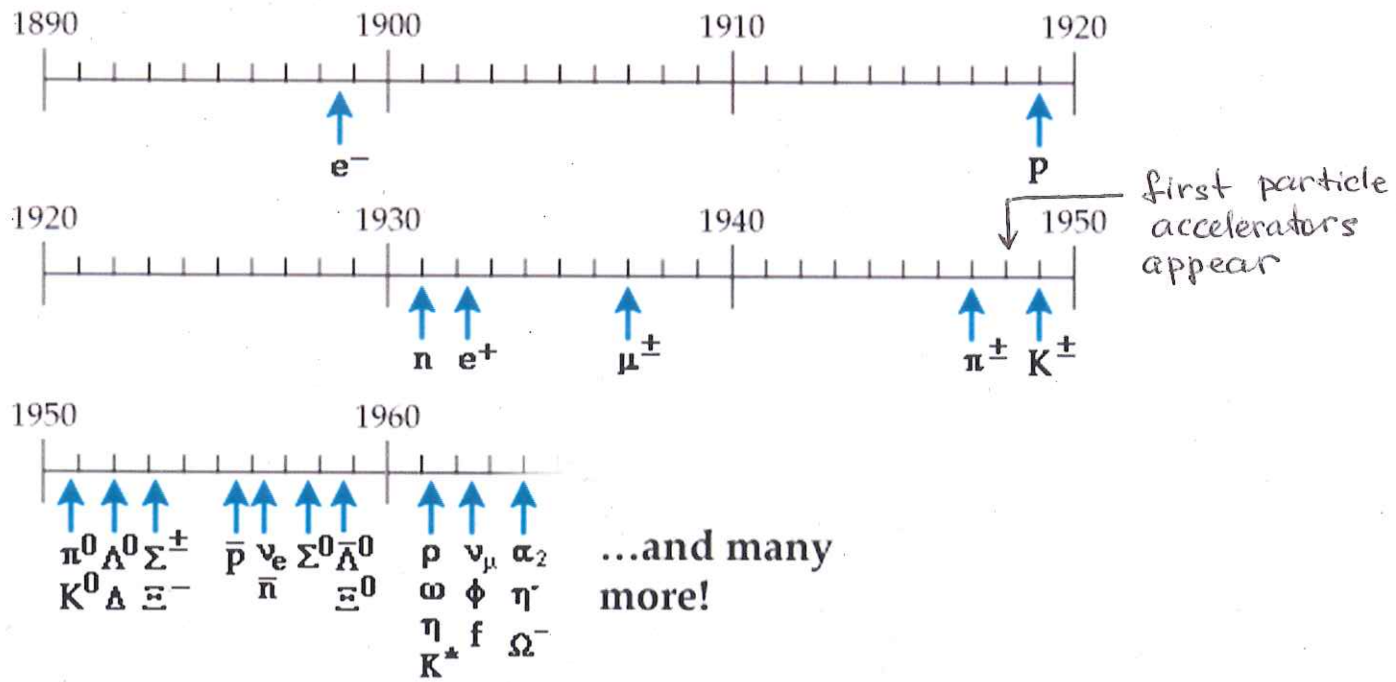
Almost stable particle: neutron n (lifetime ~ 15 min)

$n \rightarrow p + e^- + \bar{\nu}_e$ ($\bar{\nu}_e$ - anti-neutrino)
(β - decay)

People were then searching for a pion - a particle predicted to ~~parti on~~ be responsible for nuclear force
Source → cosmic rays
But they first found muon μ

Muons: μ^+ or μ^- — behave as heavy electron
unstable, lifetime ~ 2.2 μs, $m_\mu c^2 = 106$ MeV
Pions: $\pi^\pm$, π^0, lifetime ~ $3 \cdot 10^{-8}$ μs, $m_\pi c^2 = 140$ MeV

Then ~~accele~~ particle accelerators are invented in ~~1948~~ late 1940s


1890
1900
1910
1920
e⁻
p
1920
1930
1940
1950
n
e⁺
μ±
π±
K±
first particle accelerators appear
1950
1960
π⁰ K⁰
Λ⁰ Δ
Σ± Ξ⁻
p̄
νe n̄
Σ⁰
Λ̄⁰ Ξ⁰
ρ ω η K*
νμ φ f
α2 η′ Ω⁻
...and many more!

Some of the first heavy elementary particles

Kaon family K^+, K^-, K^0 lifetime $\sim 10^{-8}$ s $m_K c^2 \approx 494$ MeV

Decay: $K^+ \to \mu^+ + \nu_\mu$ (63%) $K^+ \to \pi^+ + \pi^0$

Delta family $\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$ lifetime 10^{-24} s, $m_\Delta c^2 \approx 1.23$ GeV

$\Delta^{++} \to p^+ + \pi^+$

$\Delta^+ \to \pi^+ + n^0$

$\searrow \pi^0 + p^+$

$\Delta^0 \to \pi^0 + n^0$

$\searrow \pi^- + p^+$

$\Delta^+ \to \pi^- + n^0$

Anti-particles

positron – anti-electron e^+

same mass, opposite electric charge $+e$

$$e^+ + e^- \to \left\{ \begin{array}{c} \text{total} \\ \text{annihilation} \end{array} \right\} \to \begin{array}{c} \text{pure energy} \\ \text{in a form of } \gamma \text{ quanta} \end{array}$$

We can create new ~~thaw~~ heavy particles by converting a kinetic energy into mass.

As many of these particles are unstable, and live for ~~too~~ too short period of time to detect directly, we can analize the ~~en~~ energy and momenta of the decay~~ing~~ ~~particles~~ products to figure out the mass of the original particle.

Relativistic dynamics

In the following, we consider no external forces (potential energy is zero); then total energy for any particle consists of its rest energy and its kinetic energy

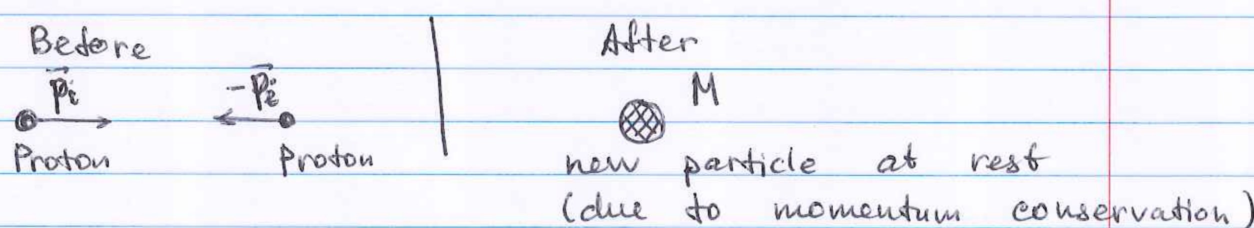
Conservation laws: energy conservation – total energy is conserved

momentum conservation – total (relativistic) momentum is conserved

Mass (rest mass) is now one of the components of the total energy and <u>is not</u> conserved.

Example: particle fusion

Two protons with $K = 2\,GeV$ collide head-on and form a new particle. What is its mass



Energy conservation: $\underbrace{E_{p_1} + E_{p_2}}_{\text{total energies of two protons}} = 2E_p = \underbrace{Mc^2}_{\text{rest energy of a new particle}}$

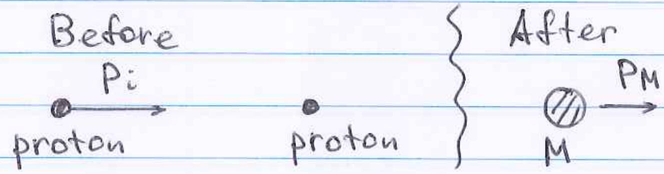
$E_p = m_p c^2 + K = 2.938\,GeV$

$Mc^2 = 2E_p = 5.88\,GeV$

$M = 5.88\,GeV/c^2$

The new mass is approx 3 times larger than the rest mass of two protons.

What mass can be achieved if only one proton is accelerated to the kinetic energy of $K_1 = 4\,GeV$, and the second one is at rest?



Momentum conservation $\vec{p}_i = \vec{p}_M$ $(p_i = p_M)$

Energy conservation $E_p + m_p c^2 = E_M$

$$(E_p + m_p c^2)^2 = E_M^2$$

$$E_p^2 + 2m_p c^2 E_p + (m_p c^2)^2 = E_M^2$$

$$E_p^2 = (p_i c)^2 + (m_p c^2)^2$$

$$E_M^2 = (Mc^2)^2 + \underbrace{(p_M c)^2}_{=(p_i c)^2}$$

$$\cancel{(p_i c)^2} + (m_p c^2)^2 + 2m_p c^2 \cdot E_p + (m_p c^2)^2 = (Mc^2)^2 + \cancel{(p_i c)^2}$$

$$(Mc^2)^2 = 2m_p c^2 E_p + 2(m_p c^2)^2$$

$$E_p = m_p c^2 + K$$

$$(Mc^2)^2 = 4(m_p c^2)^2 + 2m_p c^2 K = 4(m_p c^2)^2 \left[1 + \frac{K}{2m_p c^2}\right]$$

$$Mc^2 = 2m_p c^2 \sqrt{1 + \frac{K}{2m_p C^2}}$$

for $m_p c^2 = .938\,GeV$ and $K = 4\,GeV$

$Mc^2 = 3.32\,GeV$ (only half of what we got for a hedd-on collision)

Why? Part of the energy must go into the kinetic energy of the new particle.

Relativistic collisions

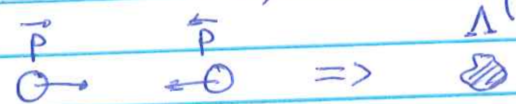
Energy is conserved $E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$

Kinetic energy $K = E - m_0 c^2$

Momentum is conserved $\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$

$E^2 = (m_0 c^2)^2 + p^2 c^2$ $\qquad \frac{c\vec{p}}{E} = \frac{\vec{v}}{c}$

Last time we considered two colliding protons forming one heavy particle. ~~There~~ Most heavy particles are unstable, and decay after a very short time



Λ (for 10^{-20} s) $\Rightarrow$ $\vec{p} + p + [p + \bar{p}]$ (a proton-anti-proton pair is created)

$\Rightarrow$ $\pi^+ + d$ (pion + deutron)

$\Rightarrow$ $p + n + \pi^+$ (proton + neutron + pion)

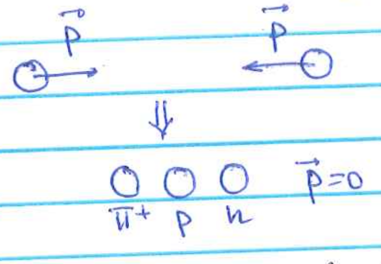
[Possible outcomes]

Suppose the last ~~reaction~~ outcome is projected

$p + p \Rightarrow p + n + \pi^+$

What is the minimum kinetic energy each initial proton must possess to make it possible?

Minimum energy $\Rightarrow$ final particles are at rest!



Initial energy $E_i = 2(m_p c^2 + K)$

Final energy $E_f = m_p c^2 + m_n c^2 + m_\pi c^2$

$\approx 2 m_p c^2 + m_\pi c^2$

$E_i = E_f \Rightarrow \cancel{2 m_p c^2} + 2K = \cancel{2 m_p c^2} + m_\pi c^2$

$K = \frac{1}{2} m_\pi c^2 = 70 \text{ MeV}$

Neutrino mystery

β-decay $\quad n \rightarrow p + e^- + \bar{\nu}_e$

Typically n & p are a part of a heavy nucleus

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}X' + e^- + \bar{\nu}_e \quad \left[{}^{14}_{6}C \rightarrow {}^{16}_{7}N + \bar{e} + \bar{\nu}_e \right]$$

n, p — stationary $\quad K \approx 0$

$m_p c^2 = 938.3$ MeV

$m_n c^2 = \cancel{836}\ 939.6$ MeV

$E_{before} = m_c c^2 \qquad E_{after} = m_p c^2 + E_{e^-}$

$E_{\bar{e}} = m_n c^2 - m_c c^2 = 1.3$ MeV $\leftarrow$ fixed value

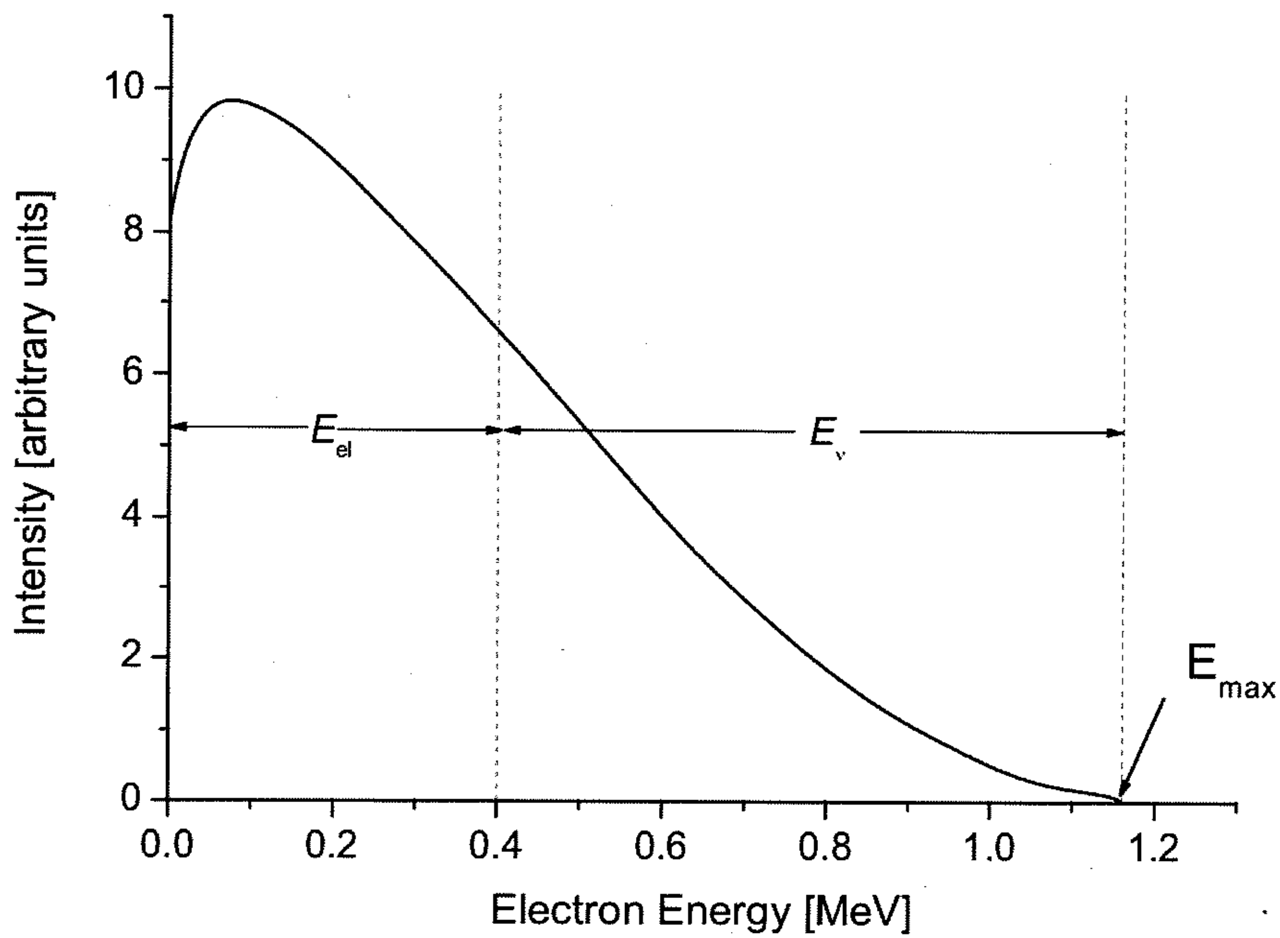
Instead the ~~spectrum~~ electrons were emitted in the range of energies!

→ That indicates that another undetectable particle is emitted — neutrino

Practically massless $\quad m_\nu \sim 0.1\ \text{eV}/c^2$

$E_\nu = c p_\nu$

$m_n c^2 - m_p c^2 = E_{\bar{e}} + E_{\bar{\nu}}$



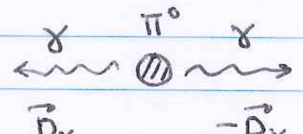
Intensity [arbitrary units]
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Electron Energy [MeV]
E_{el}
E_{ν}
E_{max}

Mass less particles — photons or gamma-quanta
γ - quanta

$m_{photon} = 0 \quad \Rightarrow \quad E = cp \quad \Rightarrow \quad v = c$

Only massless particles can move with the speed of light!

Particle decay of a neutral pion $(m_\pi c^2 = 140\,\text{MeV})$



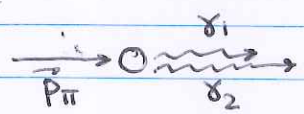
Must decay to at least two photons! (to preserve momentum conservation)

Energy conservation $\quad m_\pi c^2 = 2E_\gamma \Rightarrow E_\gamma = \frac{1}{2} m_\pi c^2 = 70\,\text{MeV}$

$$p_\gamma = \frac{E_\gamma}{c} = \frac{1}{2} m_\pi c = 70\,\text{MeV}/c$$

The situation is more complex if the pion is moving. Let's assume its kinetic energy is known $K_\pi = 200\,\text{MeV}$.
Then, the energies of two γ quanta will depend on their emission direction with respect to the direction of the original π

1. Collinear motion: γ-quanta are emitted in the same direction as π motion



Momentum conservation

$$p_\pi = p_{\gamma_1}^{(x)} + p_{\gamma_2}^{(x)}$$

Energy conservation $\qquad E_\gamma = c \cdot p_\gamma$

$$E_\pi = |p_{\gamma_1}^{(x)}|c + |p_{\gamma_2}^{(x)}|c$$

$K_\pi = 200\,\text{MeV}$

$m_\pi c^2 = 140\,\text{MeV}$

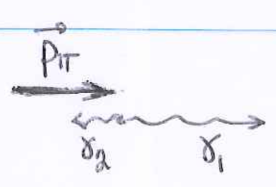
$E_\pi = 340\,\text{MeV}$

$p_\pi c = \sqrt{E_\pi^2 - (m_\pi c^2)^2} = 310\,\text{MeV}$

Only have solution if $\vec{p}_{\gamma_1} \uparrow\downarrow \vec{p}_{\gamma_2}$

$$p_\pi = p_{\gamma_1} - p_{\gamma_2} \Rightarrow cp_\pi = cp_{\gamma_1} - cp_{\gamma_2}$$

$$E_\pi = cp_{\gamma_1} + cp_{\gamma_2}$$

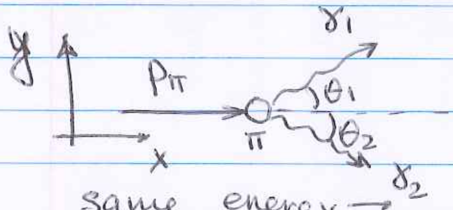


$$cp_{\gamma_1} = E_{\gamma_1} = \frac{E_\pi + cp_\pi}{2} = \frac{340\,\text{MeV} + 310\,\text{MeV}}{2} = 325\,\text{MeV}$$
(forward)

$$cp_{\gamma_2} = E_{\gamma_2} = \frac{E_\pi - cp_\pi}{2} = \frac{340\,\text{MeV} - 310\,\text{MeV}}{2} = 15\,\text{MeV}$$
(backward)

2. Can two γ-quanta have the same energy?

$E_{\gamma_1} = E_{\gamma_2} = E_\gamma \Rightarrow |\vec{p}_{\gamma_1}| = |\vec{p}_{\gamma_2}| = p_\gamma$



same energy → symmetric propagation

Momentum conservation

$$\begin{cases} x: & p_\pi = p_\gamma \cos\theta_1 + p_\gamma \cos\theta_2 \\ y: & 0 = p_\gamma \sin\theta_1 - p_\gamma \sin\theta_2 \Rightarrow \theta_1 = \theta_2 \end{cases}$$

$$p_\pi = 2 p_\gamma \cos\theta$$

Energy conservation $E_\pi = 2E_\gamma$

$$E_\gamma = \frac{1}{2} E_\pi = 170\,\text{MeV}$$

$$\cos\theta = \frac{p_\pi}{2p_\gamma} = \frac{c p_\pi}{2 c p_\gamma} = \frac{c p_\pi}{E_\pi} = \frac{310\,\text{MeV}}{340\,\text{MeV}}$$

$$\theta \approx 24^\circ$$