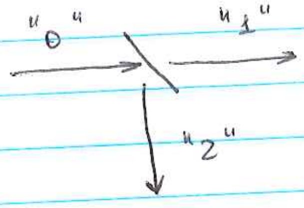
If we want to keep the quantum description consistent with the classical, we need to find an analog of electric field ~~except~~ interference, but for probabilities

wave	quantum
intensity I	probability P
electric field E	"probability amplitude"
$I \propto \|E\|^2$	ψ – wave function
	$P = \|\psi\|$



~~Af~~ Before the beam splitter the state of a particle is well-defined, it can be only in one channel "0", ~~no matter~~

After the beam splitter the particle can be either in state "1" or in state "2", but we don't know in which

Thus, the wavefunction before the beamsplitter will consist of only one contribution, describing the state "0": $\psi = \psi_0$

The probability for the particle to be in this state is 1, so $|\psi_0|^2 = 1$

However, after the beamsplitter the wavefunction of the particle must have two components

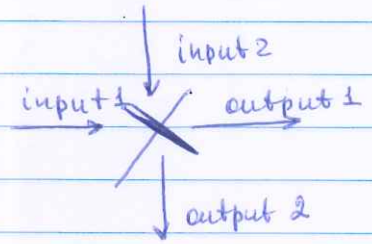
$\psi = c_1 \psi_1 + c_2 \psi_2$, where $\psi_{1,2}$ ~~cha~~ describe the quantum state ~~for~~ 1 or 2.

Probability to be in the state 1 is $P_1 = |c_1|^2$

Probability to be in the state 2 is $P_2 = |c_2|^2$

total probability $1 = P_1 + P_2 = |c_1|^2 + |c_2|^2$

Now we can figure out the rules on how the wave function changes when a light quantum hits the beam splitter

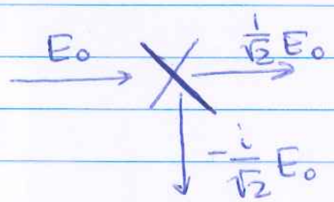


A light quanta can be in four possible states

input 1 $\longrightarrow$ ψ_0 wavefunction of the corresponding state
input 2 $\longrightarrow$ ψ_0'
output 1 $\longrightarrow$ ψ_1
output 2 $\longrightarrow$ ψ_2

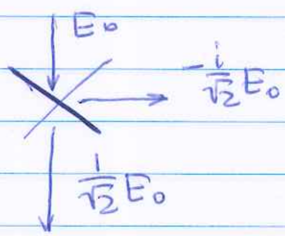
So if the wavefunction of a photon $\psi = \psi_0$, that means that a photon is in input 1 state

In analogy with the classical beamsplitter



input $\psi = \psi_0$
output $\psi = \frac{1}{\sqrt{2}}\psi_1 - \frac{i}{\sqrt{2}}\psi_2$

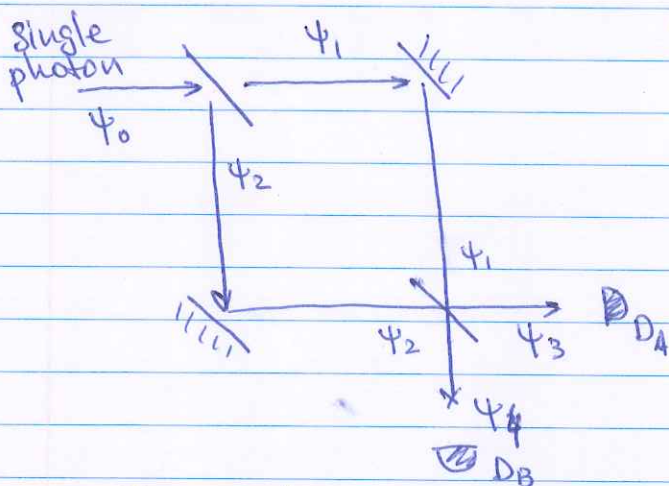
and



input $\psi = \psi_0'$
output $\psi = -\frac{i}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{2}}\psi_0$

If a photon enters one of the inputs (well-defined state) it will exits in a <u>superposition</u> of the two output states. So the photon travels in two different directions at once! At least until we measure it (then we will know for sure which pass it took, so the state will become well-defined again)

Let's see how the wavefunctions would behave in the interferometer (still $L = 2\pi n \lambda$)



Inside the interferometer

$$\psi_0 \Rightarrow \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2)$$

On the second beam splitter

$$\psi_1 \Rightarrow \frac{1}{\sqrt{2}}(\psi_4 - i\psi_3)$$

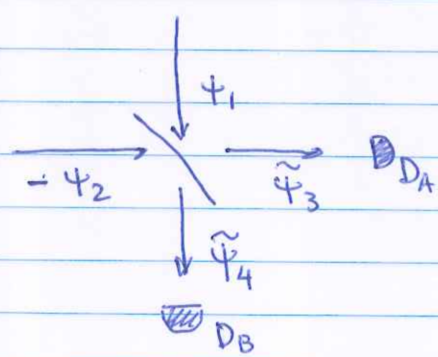
$$\psi_2 \Rightarrow \frac{1}{\sqrt{2}}(\psi_3 - i\psi_4)$$

Using this rule, we can figure out the photon's wave function after the second BS

$$\psi = \frac{1}{\sqrt{2}}\left[\cancel{\frac{1}{\sqrt{2}}\psi_4} - \frac{i}{\sqrt{2}}\psi_3 - \frac{i}{\sqrt{2}}\psi_3 - \cancel{\frac{1}{\sqrt{2}}\psi_4}\right] = -i\psi_3$$

100% probability for D_A to click, zero probability for D_B to click

If we introduce a delay again in one of the channels ($\Delta L = \lambda/2$), then $\psi_2 \to -\psi_2$



Before the second BS

$$\psi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$$

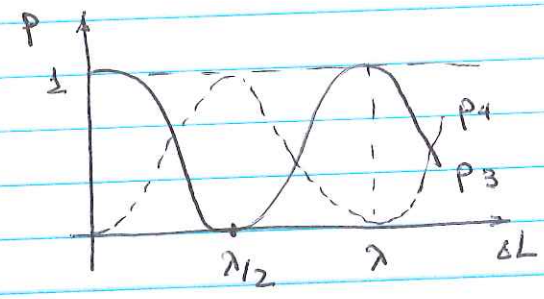
After the second BS

$$\psi = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\psi_4 - \cancel{\frac{i}{\sqrt{2}}\psi_3} + \cancel{\frac{i}{\sqrt{2}}\psi_3} + \frac{1}{\sqrt{2}}\psi_4\right] = \psi_4$$

100% probability for the detector B to click

In general, we can show that the probability to find the particle in each channel is

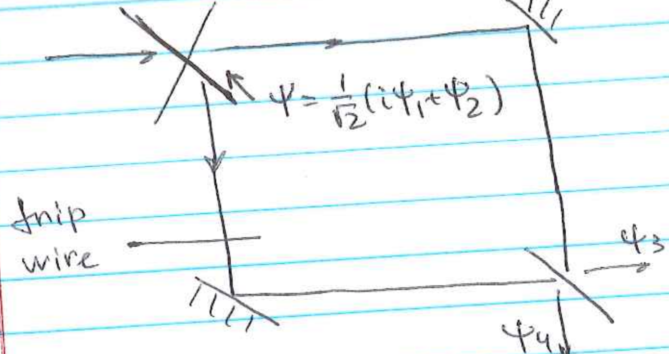
$P_3 = \cos^2\left(\frac{\pi \Delta L}{\lambda}\right)$ $P_4 = \sin^2\left(\frac{\pi \Delta L}{\lambda}\right)$



So now we have quantum and wave description giving us identical ~~de~~ prediction. However, there are some unique properties we had to introduce to make the quantum description work.

1. Wave function is not a physical thing. There is no measurements that can give us the wavefunction directly! Propabilities are measurable, but they only give absolute value squared of the coefficients. That is why the wave function is defined up to a common phase
$\frac{1}{\sqrt{2}}(\psi_1+\psi_2)$; $-\frac{1}{\sqrt{2}}(\psi_1+\psi_2)$; $\frac{i}{\sqrt{2}}(\psi_1+\psi_2)$ are all the same wavefunction

2. Making a measurement of the particle state permanently changes its state, without providing full information about it



If the tripwire fired we are certain the light pulse followed the bottom pass, so its original wave function collapses:

$\frac{1}{\sqrt{2}}(\psi_1 + i\psi_2) \Rightarrow \psi_2$

In this case interference disappears!

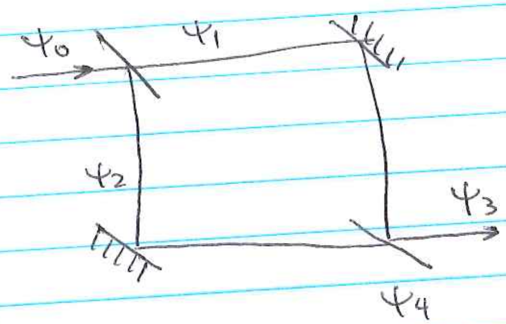
At the second BS: $\psi_2 \rightarrow \frac{1}{\sqrt{2}}(\psi_3 + i\psi_4)$

$p_3 = 1/2$, $p_4 = 1/2$

independently on the ~~phase~~ delay!

Bomb paradox

Perfectry balanced interferometer



If nothing happens, then with 100% certanty the photon emerges in the channel 3.

$p_3 = 1$, $p_4 = 0$

We put a bomb in the channel 1
If the bomb functions, it will go off for sure
So if there is no explosion, the light must follow the channel 2. In this case it splits equally at the second BS

$P_3 = 1/2$ $P_4 = 1/2$

~~So,~~ If the bomb is broken, the photon alway leaves in channel 3
If the bomb is working and ~~does~~ is not triggered, $p_3 = 0.5$ and $p_4 = 0.5$.

So: if we detect a photon in channel 4 (and no explosion) we know for sure that the bomb is working. Non-local testing!