Neutrino oscillations

Reminder – neutrinos are almost weightless particles ($m_\nu c^2 \sim 1\,eV$) that are produced in a β-decay

$$n \to p + e + \nu_e \qquad \text{electron neutrino}$$
$$+ \mu + \nu_\mu \qquad \text{muon neutrino}$$
$$+ \tau + \nu_\tau \qquad \text{tau neutrino}$$

One can measure neutrino flavor or neutrino mass. Turns out, however these are two different set of states!

To simplify, let's just look at μ and τ neutrinos. one can measure two particles of slightly different mass $\Delta m = 10^{-4}\,eV$ or two particles of different flavors.
If we choose masses as our eigenstates ψ_{m_1} and ψ_{m_2}, we can write up the flavor states as a combination of the two

$$\psi_\mu = \frac{1}{\sqrt{2}}\left(\psi_{m_1} + \psi_{m_2}\right)$$

$$\psi_\tau = \frac{1}{\sqrt{2}}\left(+\psi_{m_1} - \psi_{m_2}\right)$$

So lets assume we have a source of μ neutrinos. What happens if we let them travel?

But a little math before hand
Typical energy of neutrinos $\sim 1\,GeV$

$$E_{1,2} = \sqrt{(pc)^2 + (m_{1,2}c^2)^2} \approx pc\sqrt{1 + \frac{(m_{1,2}c^2)^2}{(pc)^2}}$$

$$\approx pc + \frac{(m_{1,2}c^2)^2}{2pc}$$

$$\boxed{\sqrt{1+x} \approx 1 + \frac{x}{2}}$$

Since m_1 and m_2 are very similar, the energies of two states are similar as well

$$E_1 - E_2 = pc + \frac{(m_1 c^2)^2}{2pc} - \left(pc + \frac{(m_2 c^2)^2}{2pc}\right) \approx$$

$$\approx \frac{1}{2pc} \cdot c^4 (m_1^2 - m_2^2) = \frac{mc^2}{pc} \cdot \Delta m c^2 \qquad (m_1^2 - m_2^2 = (m_1+m_2)(m_1-m_2))$$

~~$\Delta E \approx$~~ Since $pc \gg mc^2$ $pc \approx E$ and

$$\Delta E = \frac{mc^2}{E} \cdot \Delta m c^2$$

So now let's ~~weight~~ write down the evolution of the mass states

$$\psi_{\tau}(t) = \frac{1}{\sqrt{2}}\left(\psi_{m_1} e^{-iE_1 t/\hbar} + \psi_{m_2} e^{-iE_2 t/\hbar}\right) =$$

$$= \frac{1}{\sqrt{2}} e^{-iE_1 t/\hbar} \left(\psi_{m_1} + e^{-i\Delta E t/\hbar} \psi_{m_2}\right)$$

Note that if $\frac{\Delta E \cdot t}{\hbar} = \pi$ $e^{-i\pi} = -1$, and the state becomes $\frac{1}{\sqrt{2}}(\psi_{m_1} - \psi_{m_2})$ which is the τ state!

How far the neutrino will have to travel?

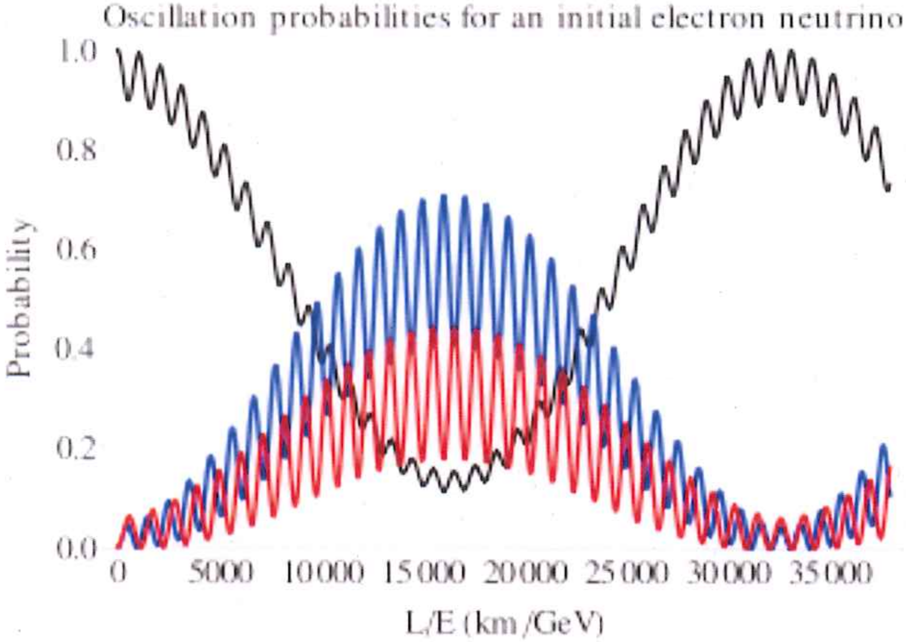
$t = L/c$ (neutrinos move at practically speed of light)

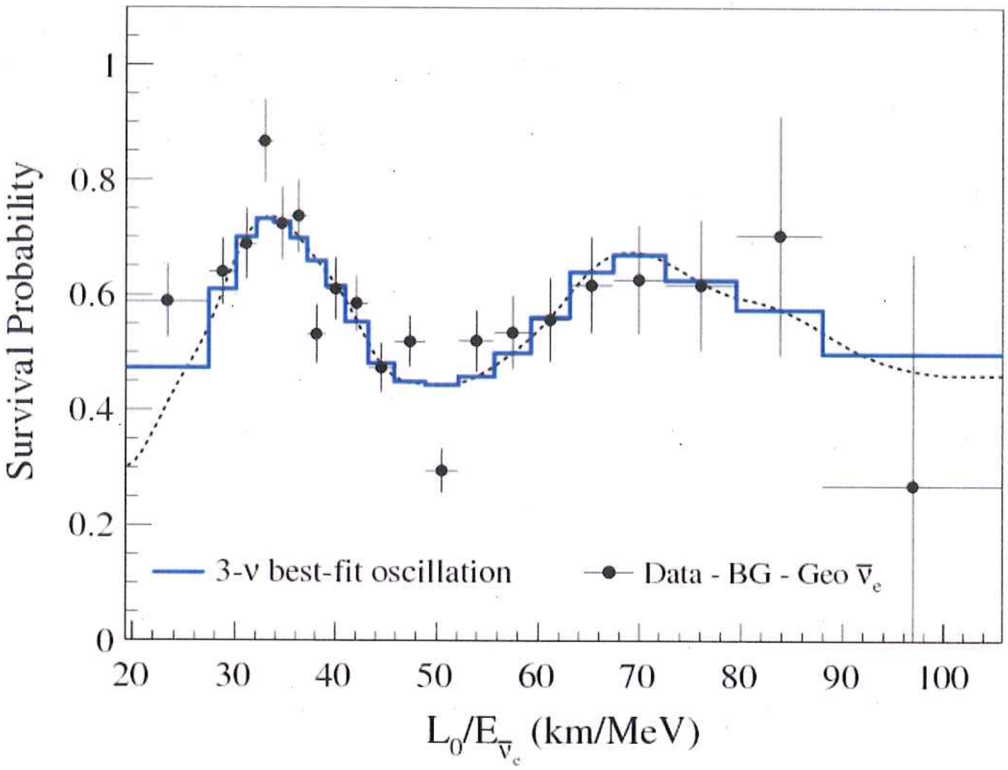
$$\frac{\Delta E \cdot L}{c\hbar} = \pi \qquad L = \frac{\pi c \hbar}{\Delta E} = \frac{\pi c \hbar}{\Delta m c^2} \cdot \frac{E}{mc^2}$$

$$c\hbar = 2 \cdot 10^{-7}\, eV \cdot m$$

$$L \sim \pi \frac{2 \cdot 10^{-7}\, eV \cdot m}{10^{-4}\, eV} \quad \frac{10^9\, eV}{1\, eV} \sim 6 \cdot 10^6\, m \sim 6000\, km$$

The best part - according to Standard Model of particle physics, ~~not~~ all neutrinos must be massless!


Oscillation probabilities for an initial electron neutrino
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
5000
10000
15000
20000
25000
30000
35000
L/E (km/GeV)



Survival Probability
1
0.8
0.6
0.4
0.2
0
20
30
40
50
60
70
80
90
100
$L_0/E_{\bar{\nu}_e}$ (km/MeV)
3-ν best-fit oscillation
Data - BG - Geo $\bar{\nu}_e$