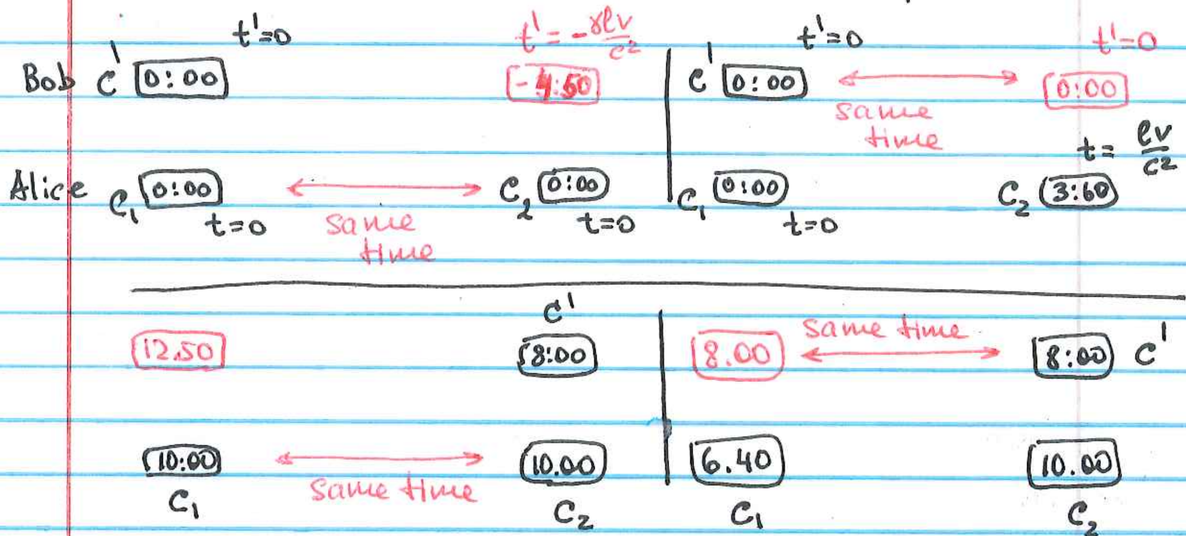
Last time we discovered that clock synchronization is hard in relativity. We can reliably synchronize two clocks only in the same RF, or if they are in the same location, but if two distant clocks are synchronized in one RF, they will not be in another.

Reminder of our 3-clock "paradox"



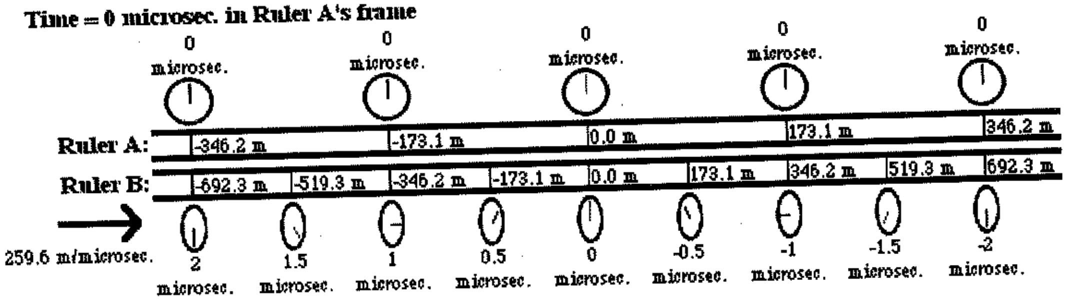
Thus, when we move between the two RFs, the time will change depend on location

Next pages display how two rulers moving along each other, with clocks attached to to each tick mark, so one can compare time readings at any point of space and time.

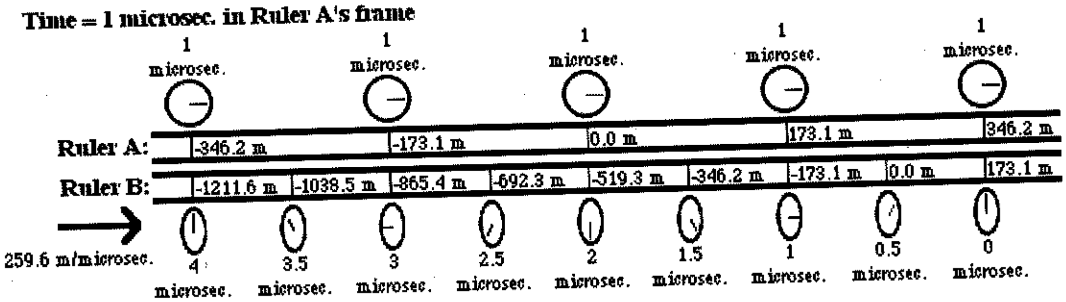
Simultaneous Events in Ruler A's Frame:

$\gamma = 2$

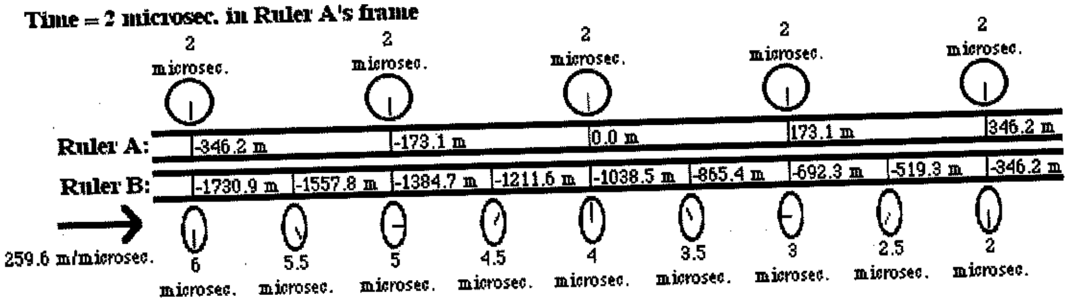
Time = 0 microsec. in Ruler A's frame



Time = 1 microsec. in Ruler A's frame

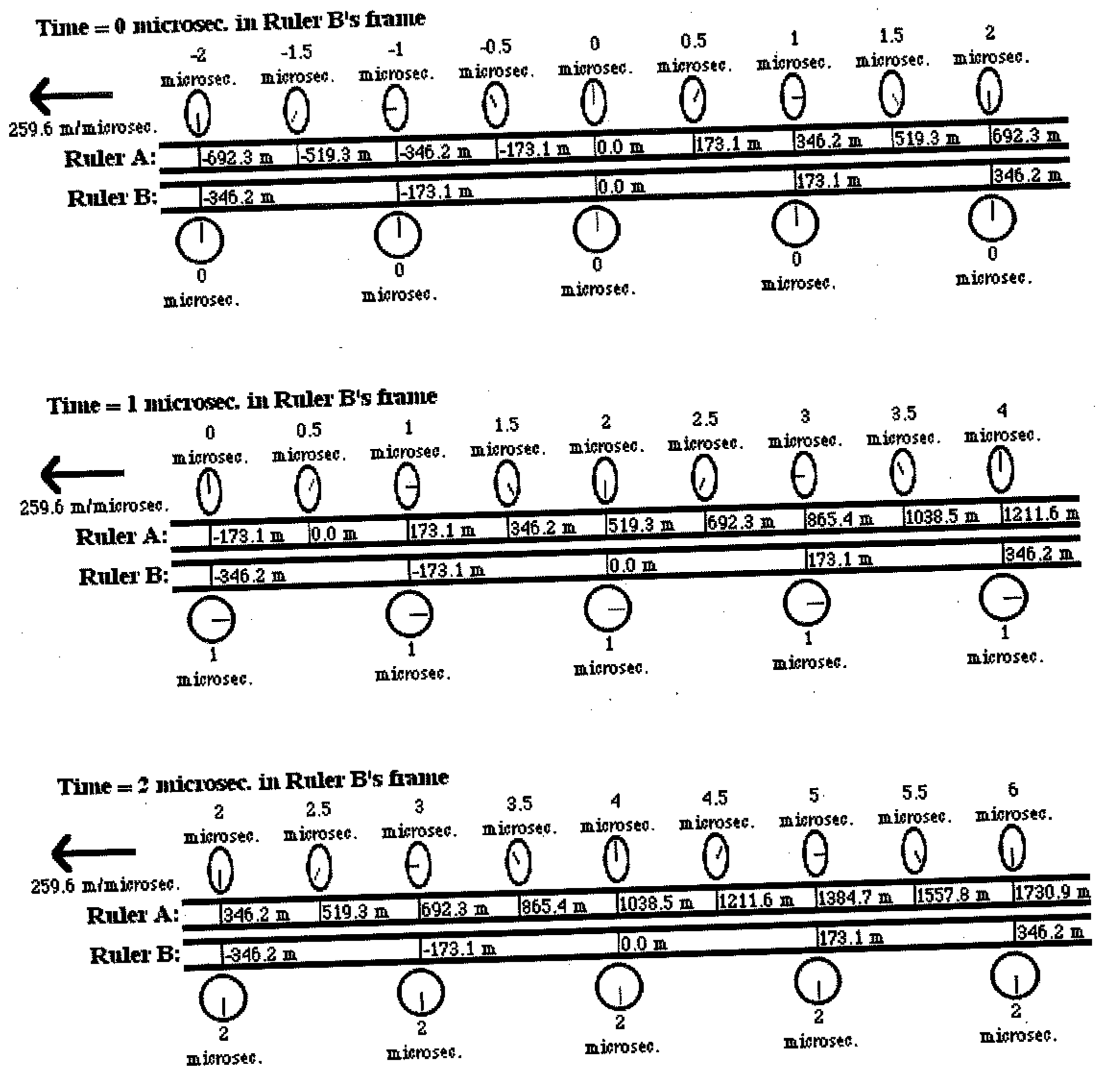


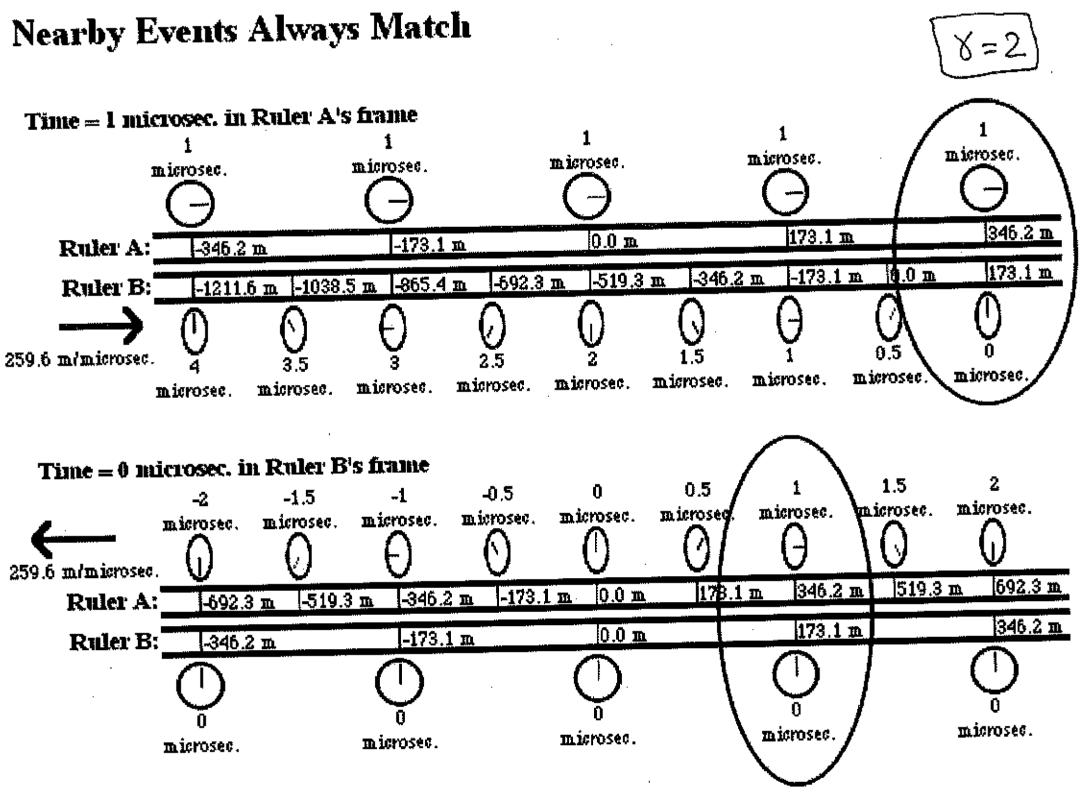
Time = 2 microsec. in Ruler A's frame



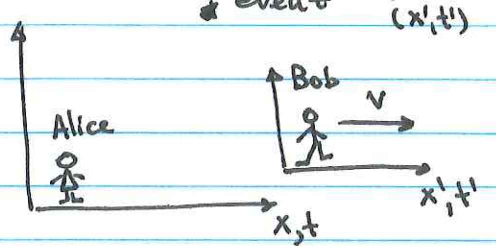
Simultaneous Events in Ruler B's Frame:

$\gamma = 2$




Nearby Events Always Match
γ=2
Time = 1 microsec. in Ruler A's frame
1 microsec.
Ruler A:
-346.2 m
-173.1 m
0.0 m
173.1 m
346.2 m
Ruler B:
-1211.6 m
-1038.5 m
-865.4 m
-692.3 m
-519.3 m
-346.2 m
-173.1 m
0.0 m
173.1 m
259.6 m/microsec.
4 microsec.
3.5 microsec.
3 microsec.
2.5 microsec.
2 microsec.
1.5 microsec.
1 microsec.
0.5 microsec.
0 microsec.
Time = 0 microsec. in Ruler B's frame
-2 microsec.
-1.5 microsec.
-1 microsec.
-0.5 microsec.
0 microsec.
0.5 microsec.
1 microsec.
1.5 microsec.
2 microsec.
259.6 m/microsec.
Ruler A:
-692.3 m
-519.3 m
-346.2 m
-173.1 m
0.0 m
173.1 m
346.2 m
519.3 m
692.3 m
Ruler B:
-346.2 m
-173.1 m
0.0 m
173.1 m
346.2 m
0 microsec.

Loretz transformation



Galeleau transformation

$$\begin{cases} x' = x - vt \\ t' = t \end{cases}$$

shoul be true for $\gamma \approx 1$

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma\left(t - \frac{v \cdot x}{c^2}\right) \end{cases} \quad \overset{v \to -v}{\Longleftrightarrow} \quad \begin{aligned} x &= \gamma(x' + vt') \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned}$$

If multiple events happen in different locations x_i at the same time t_0 in Alices RF, then their timing in Bobs RF will be different

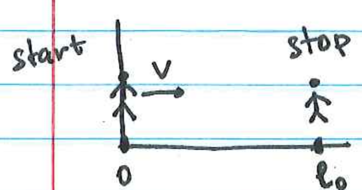
$$t_i' = \gamma\left(t_0 - \underline{\underline{\frac{v \cdot x_i}{c^2}}}\right)$$

different for different x_i

Time dilation: we are measuring the time b/w two events

		Bobs RF	Alices RF
	stop	$t_2' = \Delta t'$ $x_2' = 0$	$\boxed{t_2 = \gamma t_2'}$ $x_2 = \gamma v t_2' = v \cdot t_2$
	start	$t_1' = 0$ $x_1' = 0$	$t = 0$ $x = 0$

Length contraction



start: $x_1 = 0$, $t_1 = 0$ stop $x_2 = \ell_0$, $t_2 = \ell_0 / v$

$\underbrace{\hspace{6cm}}_{\text{Bobs position measured by Alice}}$

Bob's internal position: $x_1' = 0$, $t_1' = 0$

$$x_2' = \gamma\left(\ell_0 - \frac{\ell_0}{v} \cdot v\right) = 0$$

$$t_2' = \gamma\left(\frac{\ell_0}{v} - \frac{v\ell_0}{c^2}\right) = \frac{\ell}{v}\gamma\left(1 - \frac{v^2}{c^2}\right)$$

distance Bob sees Alice travelling $\ell' = \ell/\gamma$

$$= \frac{\ell\sqrt{1 - v^2/c^2}}{v}$$

Let's revisit our favorite clocks

$\boxed{0:00}$ $t'_1=0,\ x'_1=0$

$\boxed{0:00}$ $t_1=0,\ x_1=0$

$x'_2=\gamma\ell,\ t'_2=-\gamma\frac{\ell v}{c^2}=-4.5\,s$

$\uparrow$

$x_2=\ell,\ t_2=0$

Alices measurements are synched

or

$t'_2=0 \Rightarrow t'_2=\gamma(t_2-\frac{\ell v}{c^2})=0$

$x_2=\ell$

$\Downarrow$

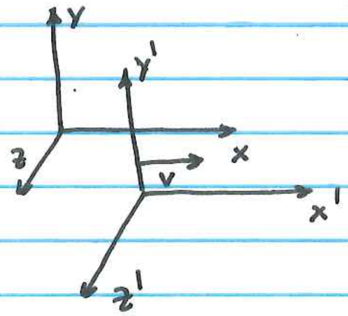
$t_2=\frac{\ell v}{c^2}=3.6\,s$

Bob's measurements are synched

It is important to remember that Lorentz transformation provide the rules to convert the time-space coordinates of individual events b/w two RFs. Often, this is not the most convenient method, since some measurements require multiple events, and sometimes it is not convenient to define the coordinates for each event, involved in the measurement. Then using the concepts of time dilation and length contraction is easier.

Relativistic velocity addition

$\vec{u}\,(u_x, u_y, u_z)$
(u'_x, u'_y, u'_z)
velocity of an object as measured in two RF



Galelean velocity addition

$$\vec{u}' = \vec{u} - \vec{v}$$

or $\begin{cases} u'_x = u_x - V \\ u'_y = u_y, \quad u'_z = u_z \end{cases}$

Lorentz transformations

$$\begin{cases} x' = \gamma(x - vt) \\ y' = y, \quad z' = z \\ t' = \gamma\left(t - \frac{xv}{c^2}\right) \end{cases}$$

$u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt}$

$u'_x = \frac{dx'}{dt'}$

$$dx' = \gamma(dx - v \cdot dt)$$
$$dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

$$u'_x = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx} = \frac{u_x - V}{1 - \frac{V \cdot u_x}{c^2}}$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - \frac{V}{c^2}dx\right)} = \frac{u_y}{\gamma\left(1 - \frac{V u_x}{c^2}\right)}$$

depends on u_x!

$$\begin{cases} u'_x = \dfrac{u_x - V}{1 - \frac{V u_x}{c^2}} \\[2ex] u'_y = \dfrac{u_y}{\gamma\left(1 - \frac{V u_x}{c^2}\right)} \\[2ex] u'_z = \dfrac{u_z}{\gamma\left(1 - \frac{V u_x}{c^2}\right)} \end{cases} \quad \overset{V \to -V}{\Longleftrightarrow} \quad \begin{cases} u_x = \dfrac{u'_x + V}{1 + \frac{V u'_x}{c^2}} \\[2ex] u_y = \dfrac{u'_y}{\gamma\left(1 + \frac{V u'_x}{c^2}\right)} \\[2ex] u_z = \dfrac{u'_z}{\gamma\left(1 + \frac{V u'_x}{c^2}\right)} \end{cases}$$

If $V = 0.9c$ and $u_x = -0.9c$?

$$u'_x = \frac{0.9c + 0.9c}{1 + \frac{(0.9c)^2}{c^2}} = 0.994c \quad (\text{not } 1.8c)$$