Wave-particle duality of light

Sometimes we can describe light as an electromagnetic wave and sometimes as a stream of massless particles, carried well-defined energy and momentum.

EM wave

$$\vec{E} = \vec{E}_0 \cos\left(\frac{2\pi}{\lambda} z - 2\pi f \cdot t + \varphi\right)$$

Intensity $\propto |E_0|^2$

Particles (photons)

$$E = hf \quad |\vec{P}| = h/\lambda \quad f = c/\lambda$$

Intensity $\propto n \cdot hf$

~~Wave physicists describe oscillations phases.~~

Steady-state wave: its amplitude and phase do not change

$$E = \underbrace{E_0}_{\text{constant in time}} \cos\left(\underbrace{\frac{2\pi}{\lambda} z - 2\pi f \cdot t}_{\text{oscillating part}} + \varphi\right)$$

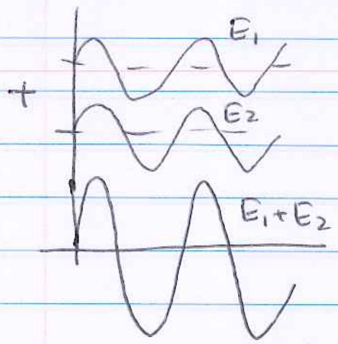
For interference effects the phase is very important

$$E_1 = E_0 \cos\left(\frac{2\pi}{\lambda} z - 2\pi f \cdot t\right)$$

$$E_2 = E_0 \cos\left(\frac{2\pi}{\lambda} z - 2\pi f \cdot t + \varphi\right)$$

φ will determine how the two waves add up.

$\varphi = 0$

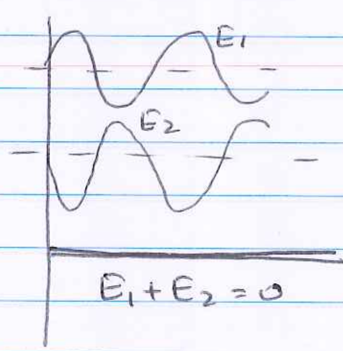


total amplitude

$E_\Sigma = E_1 + E_2$

constructive interference

$\varphi = \pi\ (180°)$



destructive interference

How physicist prefer to describe oscillations /waves

Euler relation $e^{i\varphi} = \cos\varphi + i\sin\varphi$

Instead of using sin and cos, it is almost always more convenient to use a complex exponent

$\frac{d}{dx}\cos x = -\sin x$ $\qquad$ vs $\qquad$ $\frac{d}{dx} e^{ix} = i \cdot e^{ix}$

$\frac{d}{dx}\sin x = \cos x$

So instead of using $E(z,t) = E_0 \cos\left(\frac{2\pi}{\lambda}z - 2\pi f t + \varphi\right)$

we can use

$$E(z,t) = \mathrm{Re}\left[E_0 e^{i\left(\frac{2\pi}{\lambda}z - 2\pi f t + \varphi\right)}\right]$$

or we don't even write Re[...] and remember to take it at the very end.

So instead $E(z,t) = \underbrace{E_0 e^{i\varphi}}_{\text{complex amplitude.}} \cdot \underbrace{e^{i 2\pi/\lambda z - i2\pi f t}}_{\text{oscillating part, does not change}}$

$$\underset{\text{reference}}{E_0} \xrightarrow{+\varphi = \pi/2} E_0 e^{i\pi/2} = iE_0 \xrightarrow{+\varphi = \pi/2} E_0 e^{i\pi} = -E_0$$

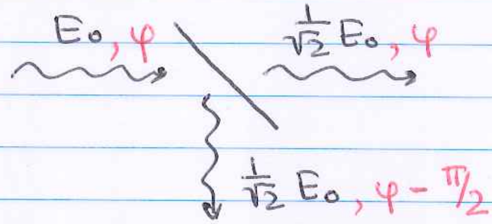
iE_0: a wave phase-shifted by $\pi/2 = 90°$

$-E_0$: a wave phaseshifted by $\pi = 180$

with respect to the reference.

Beam splitter (semitransparent mirror)
Visually – ~~two beams~~ one beam is split into two beams of equal intensity

Wave description

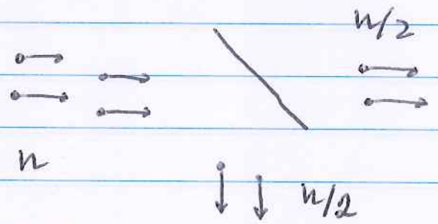


$I \propto |E_0|^2$

$\frac{1}{2} I \rightarrow \frac{1}{\sqrt{2}} E_0$

If we use complex amplitudes $E_0 e^{i\varphi}$ splits into a transmitted wave $\frac{1}{\sqrt{2}} E_0 e^{i\varphi}$ and a reflected wave $\frac{1}{\sqrt{2}} E_0 e^{i(\varphi - \pi/2)} = -\frac{i}{\sqrt{2}} E_0 e^{i\varphi}$

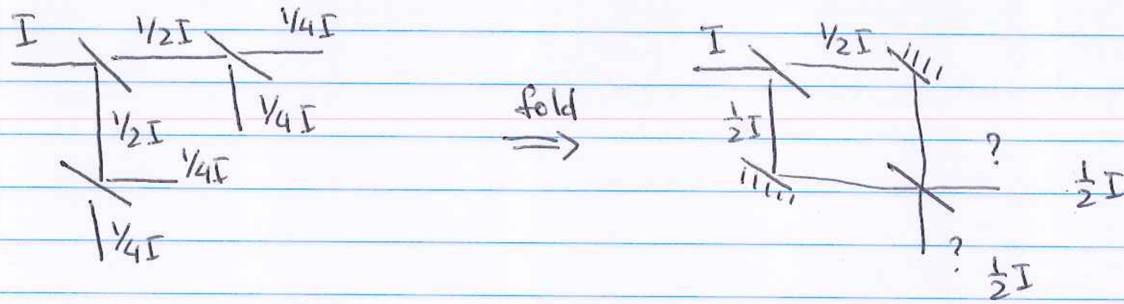
Particle description

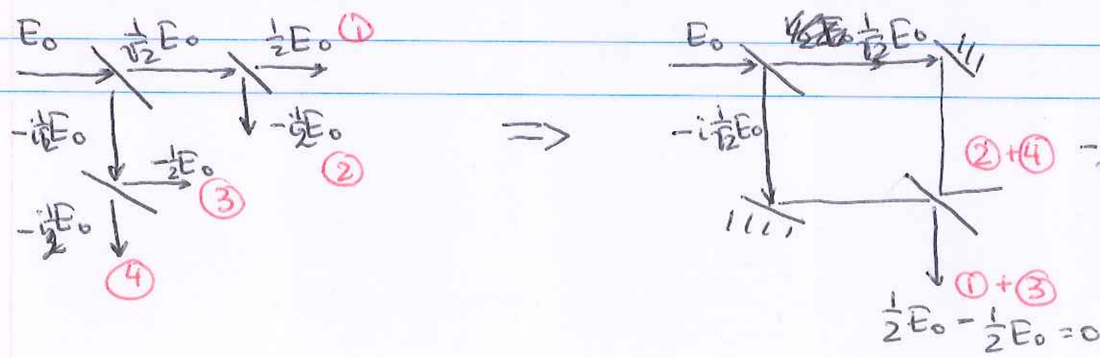


50% probability to get transmitted
50% probability to get reflected

No phase information?

A series of beam splitters





$-\frac{i}{\sqrt{2}} E_0 - \frac{i}{\sqrt{2}} E_0 = -iE_0$ full intensity