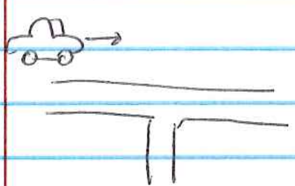
Classical vs Quantum description of the world

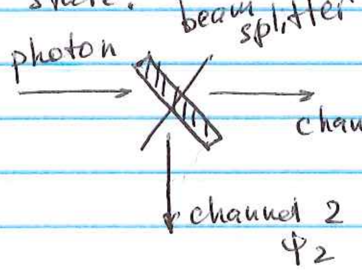
Classical physics is determenistic, ~~at~~ ~~p~~ which means that if the experiment is repeated exactly many times, all the outcomes are the same

Car at the intersection will go either streight or right, and even if we don't know ~~to~~ what the drive decided to do, the act of measurement will not change its state.



So the act of measurement does not change the state of the object. Consequently, a complete information about all the parameters of the system can be obtained.

In quantum physics we describe the state of the object using its wave function, and in many cases we can do this precisely. However, some measurements can alter the quantum state.



After the beamsplitter the state of the photon is

$$\psi_0 = \frac{1}{\sqrt{2}}\psi_1 - \frac{i}{\sqrt{2}}\psi_2$$

so if measured, a photon probabilistically can be found in either channel. However, if we do an undestructive measurement, and, say, discover the photon in the channel 1, then any ~~consequenti~~ following measurements will always show ~~parti~~ the object in that channel.

So

before the measurement

$$\psi_0 = \frac{1}{\sqrt{2}}\psi_1 - \frac{i}{\sqrt{2}}\psi_2 \xrightarrow{\quad} \text{measure!} \quad \begin{cases} \xrightarrow{\quad} \psi_0 = \psi_2 & \left(\begin{matrix}50\% \\ \text{cases}\end{matrix}\right) \\ \text{or} \\ \xrightarrow{\quad} \psi_0 = \psi_1 & \left(\begin{matrix}50\% \\ \text{cases}\end{matrix}\right) \end{cases}$$

The state of the object has changed!

~~Not~~ We can often choose the states so that they can be labeled by the value of a certain parameter.
For example, in the beam splitter case, we chose two states ψ_1 and ψ_2 so that they correspond to a particular ~~dirs~~ path after the splitter.
If an object is in either of these states, the measurement of which-path will always give the same answer. However, in an object in in the combination of the two, the same measurement will alter the object's state, changing it ~~the~~ to either ψ_1 or ψ_2, depending on the measurement outcome.

In many future problems we will be looking for states that have ~~destined~~ the defined values of the total energy (energy levels).
We will see in a second that if the motion of the particle is restricted to a certain range of ~~coord.~~ space, ~~then~~ its values of possible total energies corresponding to stationary states will be quantized (i.e only certain values of total energy are possible).

Before we start calculating wavefunctions and energy levels for specific situations, we can qualitatively investigate why the energy is quantized.

Photon wavefunction $\psi(x,t) = \frac{1}{\sqrt{2\pi}} e^{i\frac{2\pi}{\lambda}x - i2\pi f t}$

Wave-particle duality $\lambda = \frac{2\pi\hbar}{p} \Rightarrow \frac{2\pi}{\lambda} = \frac{p}{\hbar}$

$$2\pi\hbar f = E \Rightarrow 2\pi f = E/\hbar$$

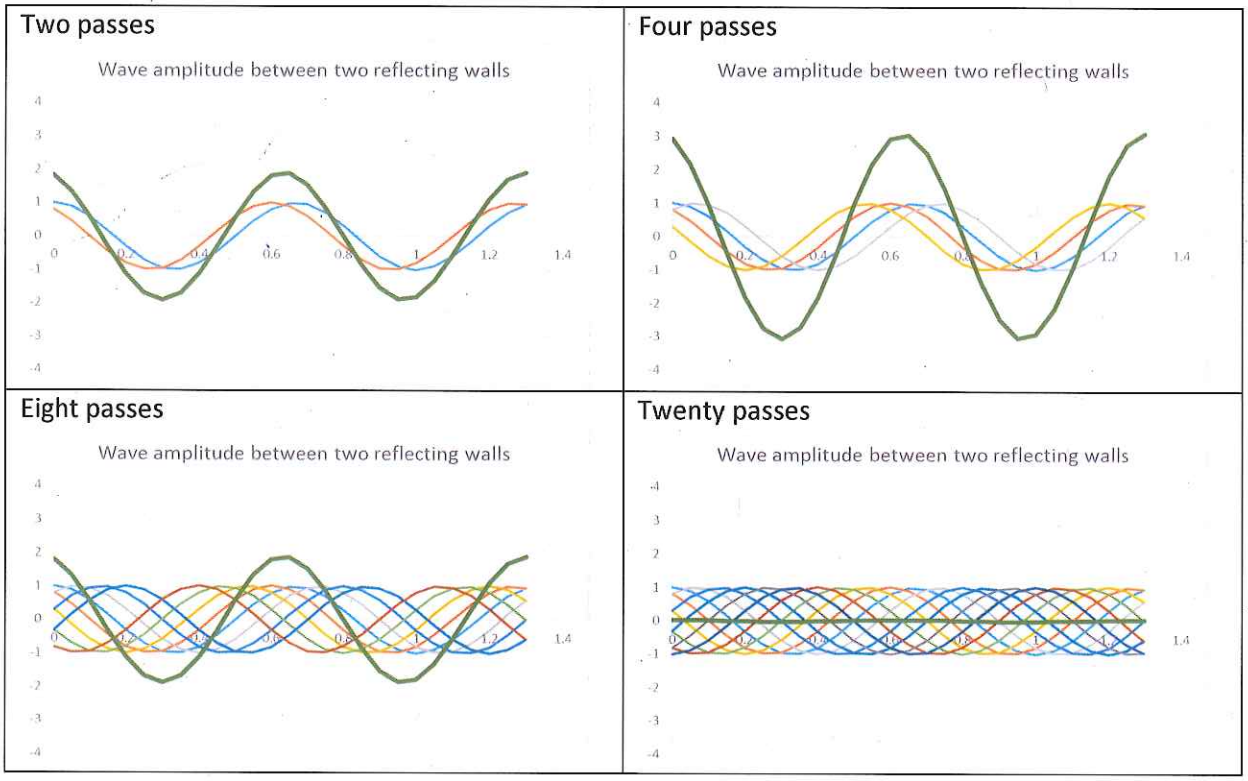
For a free moving particle with well-defined energy and momentum along x, the wave function is $\psi(x,t) = \frac{1}{\sqrt{2\pi}} e^{ipx/\hbar - iEt/\hbar}$

This is the analog of a steady-state light wave: apart from the oscillating phase, nothing changes in time, and the particle is completely delocolized in space (x) and tim (t)

Now suppose ~~that~~ such particle is placed b/w two ~~walls~~ unpenetrable walls. ~~For the particle~~ Then the particle can only bounce back and forth b/w the walls. If the size of the ~~a~~ allowed region is randomly chosen, the total field will be ~~zero~~ averaged to zero. To allow for a steady state oscillation to exist inside, we have to create the condition for a standing wave: $2L = n\lambda \Rightarrow \lambda_n = \frac{2L}{n}$

Thus the momentum of the particle can only have specific values $p_n = \frac{2\pi\hbar}{\lambda_n} = \frac{\pi\hbar n}{L}$

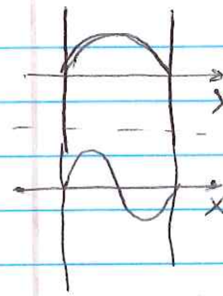
and the kinetic energy = total energy $E_n = \frac{p_n^2}{2m} = \frac{\pi^2\hbar^2 n^2}{2mL^2}$


Two passes
Wave amplitude between two reflecting walls
4 3 2 1 0 -1 -2 -3 -4
0 0.2 0.4 0.6 0.8 1 1.2 1.4
Four passes
Wave amplitude between two reflecting walls
4 3 2 1 0 -1 -2 -3 -4
0 0.2 0.4 0.6 0.8 1 1.2 1.4
Eight passes
Wave amplitude between two reflecting walls
4 3 2 1 0 -1 -2 -3 -4
0 0.2 0.4 0.6 0.8 1 1.2 1.4
Twenty passes
Wave amplitude between two reflecting walls
4 3 2 1 0 -1 -2 -3 -4
0 0.2 0.4 0.6 0.8 1 1.2 1.4

Thus, the energy of the particle will be precisely defined: ~~for~~

$$E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$$

and each value of the energy will correspond to a specific standing wave with the wavelength $\lambda_n = 2L/n$



Lowest energy $n=1$ $E_1 = \dfrac{\pi^2\hbar^2}{2mL^2}$ $\lambda_1 = 2L$

$n=2$ $E_2 = \dfrac{2\pi^2\hbar^2}{mL^2}$ $\lambda_2 = L$

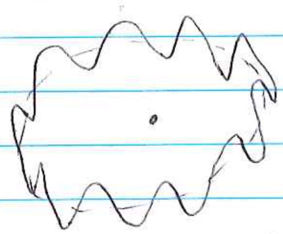
and so on

We can verify the uncertainty principle:
$\Delta x \sim L$ (particle is localized inside the walled region)
Momentum $p_x = \pm p_n = \pi\hbar n/L$, so $\Delta p_x \sim \pi\hbar n/L$

$$\Delta x \cdot \Delta p \sim \cancel{L} \cdot \frac{\pi\hbar n}{\cancel{L}} = \pi\hbar \cdot n$$

so for any n $\Delta x \cdot \Delta p \gtrsim \pi\hbar$

Similar idea will be applied for the electron motion around the nucleus:



Electron's wavefunction must form a 3D standing wave, and that will dictate the possible values of electron's energy. We will calculate it later, since the 3D math is much more complex