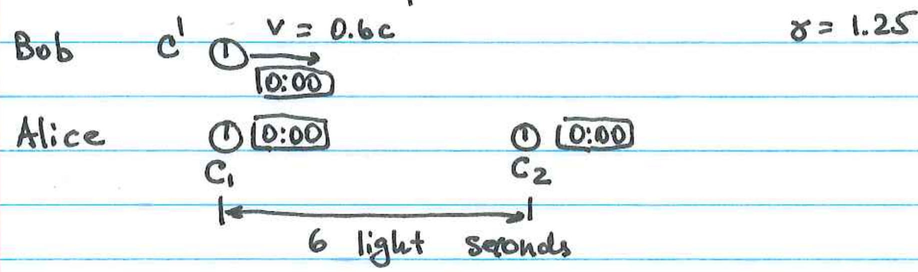
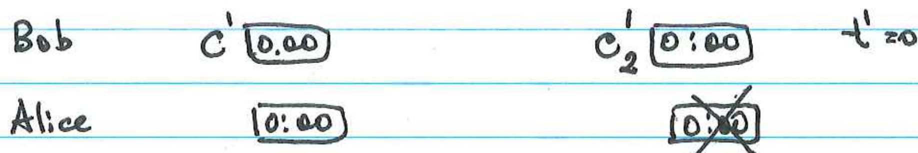
Three - clock "paradox"



According to Alice, it takes Bob 10 s to get from c_1 location to c_2 location
So when he gets there the clocks read

Bob's clock run slower due to the time dilation $t' = t_0/\gamma = 10s/1.25 = 8s$

However, Bob sees 10s on Alice's clock c_2, so does that mean the relativity is broken?
No. Because his measurement is not properly done! He did not check the reading on c_2 when he observed c_1 at $t'=0$. To do the measurements correctly, he would need another clock (in his RF) at c_2 location

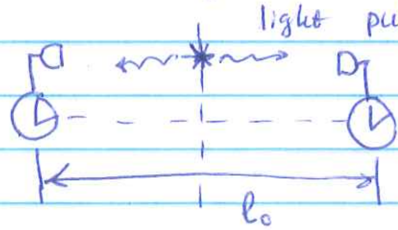


at relativistic speeds the clock synchronization is not universal

How to synchronize two distant clocks?

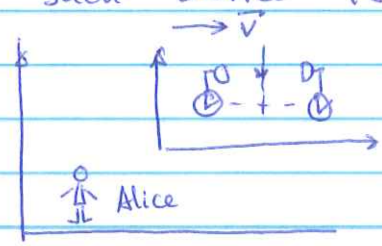
Speed of light is constant regardless of the RF, so if ~~an~~ two ~~source~~ clocks flash light that reaches an observer located precisely in between at the same time — the clocks are synchronized!

Clock synchronization device:



Time it takes for the light to travel $\Delta t_0 = \ell/2c$ in the clock's rest frame.

If such device is moving

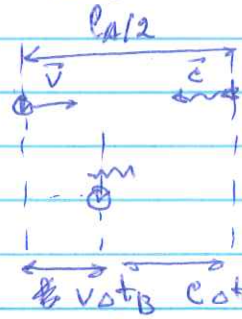


1. The length of Bob's apparatus is shoreter $\ell_A = \ell_0/\gamma = \ell_0/\sqrt{1-v^2/c^2}$

2. The clocks are moving as light pulses propagate!
   Back clocks moves toward the light pulse, and the front clocks moves away from its light pulse: $\Delta t_{Back} < \Delta t_{front}$!

Back clocks: $t=0$

$t = \Delta t_{Back}$

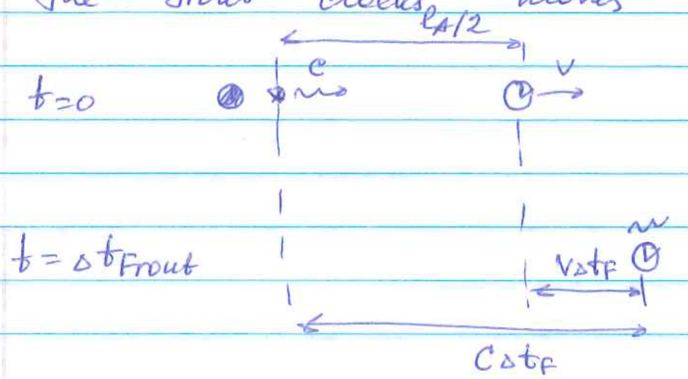


$$v\Delta t_B + c\Delta t_B = \ell_{A/2}$$

$$\Delta t_B = \frac{\ell_A}{2(v+c)} = \frac{\ell_0}{2\gamma(v+c)}$$

According to Alice, that's how much time passes from the original flash to the back clock starting.

The front clocks moves away



$$c\Delta t_F - v\Delta t_F = l_A/2$$

$$\Delta t_F = \frac{l_A/2}{c-v} = \frac{l_0}{2\gamma(c-v)}$$

Clearly $\Delta t_F > \Delta t_B$ for any $v>0$.

Thus, according to Alice, Bob's clock did not started at the same time!
The back clock has started first, and the front clocks started later

$$\Delta t_F - \Delta t_B = \frac{l_0}{2\gamma}\left(\frac{1}{c-v} - \frac{1}{c+v}\right) = \frac{2v}{c^2-v^2}\cdot\frac{l_0}{2}\sqrt{1-v^2/c^2}$$

$$\Delta t_F - \Delta t_B = \frac{vl}{c^2}\frac{1}{\sqrt{1-v^2/c^2}}$$ (measured in Alice's reference frame)

Let's imagine Alice takes simultaneous pictures of Bob's two clocks at the moment the front clocks is a just starting?
(We assume she has two cameras strategically plased exactly next to the two Bob's clocks, as they pass by in the right moment)

Back — has been running for awhile

Front — just started! $t_F=0$

$\frac{vl}{c^2}\frac{1}{\sqrt{1-v^2/c^2}}$ in Alice's reference frame

$\boxed{\frac{vl}{c^2}}$ in Bob's reference frame (his ticks are slower)

Wait a moment: in Bob's frame the two clocks are perfectly synchronized!
Why the pictures show different time?!

That is because Alice's cameras are synchronized only in her RF, not in Bob's.
Repeating exact same discussion from Bob's point of view, we can show that according to Bob, the front camera fired first, and then after time vl/c^2 the back camera took its picture.

Bottom line: if clocks are synchronized in one reference frame, they will not appear so from any moving RF.
(Two ~~e~~ events in different locations ~~at~~ may be simultaneous only in one RF, but will be registered as occuring at different ~~the~~ times in any other.)

So lets look back on our three-clock problem:

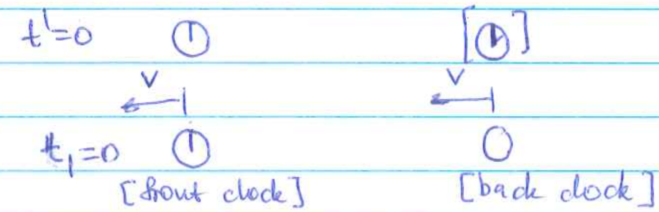
~~Then~~ At $t=0$ all three clocks show the same time.

$t'=0$

$t=0$ $\quad$ $t=0$

But if Bob had a second clocks at the distant location, ~~the~~, Alice would not see them showing $t=0$!

Lets check the situation from Bob's RP



According to Bob, the back clocks are ahead by

$$\frac{v \cdot l}{c^2} = \frac{0.6c \cdot 6c \cdot s}{c^2} = 3.6s$$

So ~~when~~ when Bob started his shuttle clocks, and C_1 showed $t=0$, the C_2 clocks were already displaying $3.6s$

Thus, when the shuttle arrives to C_2 location, and sees $10s$ reading on that clocks, in $8s$, Bob concludes that only $10s - 3.6s = 6.4$ seconds ~~to tick~~ in Alices frame, which is less than his 8-second trip. Time dilation!

C' 0:00	0.00	8.00	C'_1 8.00
C_1 0:00	C_2 3.60	6.40 C_1	C_2 10.00

Follow-up question:

Let's imagine that at the instant Bob's shuttle passes the C2 clock, he turns around and take an instant photograph of the C1 clock (with his super-fancy future smartphone with Hubble-telescope-like optics). What time the C1 clock would display on that picture?