Phys 201-2017F
Tutorial
waves / Michelson

Waves (brief reminder)

$$\vec{E} = E_0\, \vec{e} \cos(\underbrace{\vec{k}\vec{r} - \omega t + \varphi}_{\text{phase}})$$

E_0 – amplitude $\quad\quad$ $\vec{e}$ – direction of polarization

$\vec{k}$ → wave vector, $k = \frac{2\pi}{\lambda}$ $\quad$ λ – wavelength
shows direction of propagation

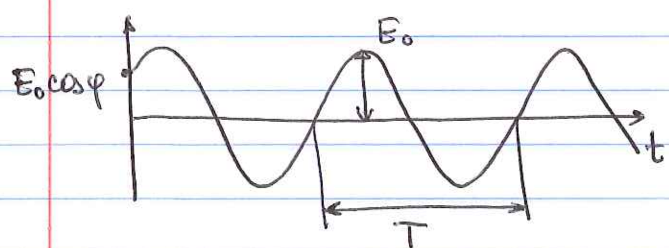
ω – frequency $\quad$ $\omega = 2\pi f$
$\quad\quad$ [angular] [just frequency]

$\omega = \frac{2\pi}{T} = 2\pi c/\lambda$

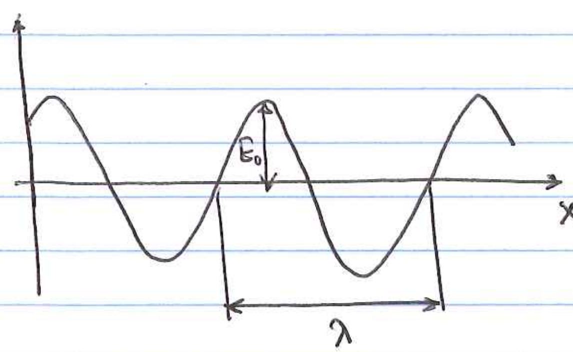
T – period $\quad\quad$ c – speed of light / sound / wave

φ – phase offset

Snapshot in time at the same position $\vec{r} = 0$



Snapshot in space at the same time $t=0$



Since c is usually defined by the nature of the wave, and

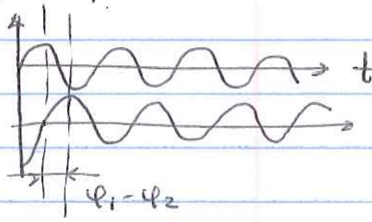
$$\lambda = \frac{2\pi c}{\omega}, \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

then waves of same frequency have same wavelength.

Interference = superposition of two waves of same polarization, frequency and direction

$E_1 \cos(kz - \omega t + \varphi_1)$

$E_2 \cos(kz - \omega t + \varphi_2)$



For simplicity assume

$E_1 = E_2 = \frac{1}{2} E_0$

Useful identity: $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$$E_{tot} = \frac{1}{2}E_0 \cos(kz - \omega t + \varphi_1) + \frac{1}{2}E_0 \cos(kz - \omega t + \varphi_2) =$$

$$= E_0 \cos\left(kz - \omega t + \frac{\varphi_1 + \varphi_2}{2}\right) \cdot \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right)$$

The result is the wave of the same frequency, and with the amplitude $E_0 \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right)$

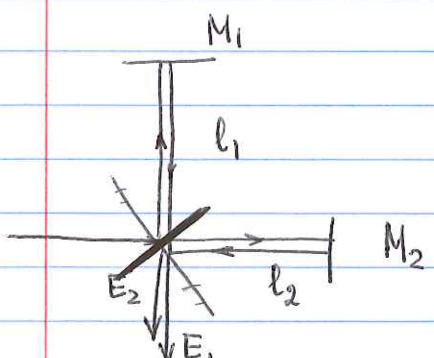
If $\cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) = \pm 1$ – constructive interference

$\varphi_1 - \varphi_2 = 0, \pm 2\pi, \ldots \quad 2\pi n \quad n = 0, 1, 2 \ldots$

If $\cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) = 0$ – destructive interference

$\varphi_1 - \varphi_2 = \pm\pi, \pm 3\pi, \ldots \quad \pi \pm 2\pi n \quad n = 0, 1, 2 \ldots$

Michelson interferometer



After two beams are recombined on the beam-splitter (after reflections)

$E_1 = \frac{1}{2} E_0 \cos(kz - \omega t + \varphi_1)$

$\varphi_1 = k \cdot 2l_1 = \frac{2\omega}{c} l_1$

$E_2 = \frac{1}{2} E_0 \cos(kz - \omega t + \varphi_2)$

$\varphi_2 = k \cdot 2l_2 = \frac{2\omega}{c} l_2$

So depending on the difference in the length of two arms, we will see constructive or destructive interference at the output

$$\varphi_1 - \varphi_2 = \frac{2\omega}{c}(l_1 - l_2) = \frac{4\pi}{\lambda}(l_1 - l_2)$$

Suppose that $l_1 = l_2 \Rightarrow \varphi_1 - \varphi_2 = 0$
constructive interference

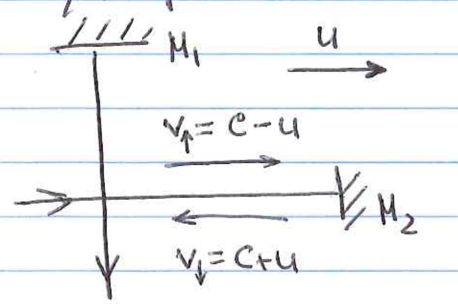
if now $l_1 - l_2 = \lambda/4$ (a few hundred microns for visible light)
bright output changes to the dark one

Michelson interferometer is an amazing tool for measuring small displacements

Aether measurement.

If the aether idea is correct, then the speed of light is c only at the aether's rest frame. For any other situations we need to take into account the "aether drag".

Assume the experiment (Michelson interferometer) moves with speed u w/ respect to the aether.

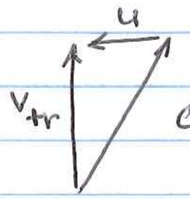


(aether is assume to be stationary)

For the second arm:

$$\varphi_2 = \frac{l_2\omega}{v_\uparrow} + \frac{l_2\omega}{v_\downarrow} = \frac{l_2\omega}{c-u} + \frac{l_2\omega}{c+u} = \frac{2cl_2\omega}{c^2 - u^2}$$

For the first arm (transverse drag)



$$V_{tr} = \sqrt{c^2-u^2}$$

$$\varphi_1 = \frac{2l_1\omega}{V_{tr}} = \frac{2l_1\omega}{\sqrt{c^2-u^2}}$$

Conditions for destractive (or constructive) interference depend on the ~~aether~~ orientation of the experiment motion w/ respect to ~~the~~ its direction in the aether "bath". Assuming $l_1 = l_2$

$$\varphi_1 - \varphi_2 = \frac{2l}{\sqrt{c^2-u^2}} - \frac{2cl}{c^2-u^2} = \frac{2\omega l}{c}\left[\frac{1}{\sqrt{1-u^2/c^2}} - \frac{1}{1-u^2/c^2}\right]$$

using Taylor expansion (for $u/c \ll 1$)

$$\frac{1}{\sqrt{1-u^2/c^2}} \approx 1 + \frac{u^2}{2c^2} \qquad \frac{1}{1-u^2/c^2} \approx 1 + \frac{u^2}{c^2}$$

$$\varphi_1 - \varphi_2 = \frac{2\omega l}{c}\left(-\frac{u^2}{2c^2}\right)$$

For $u \approx 3\cdot 10^{+4}$ m/s and $c \approx 3\cdot 10^{8}$ m/s relative shift of the dark / bright fringe $\simeq 5\cdot 10^{-9}$

$$\frac{\varphi_1 - \varphi_2}{\pi/2} = \frac{2}{\cancel{\pi}}\,\frac{\cancel{2}\cancel{\pi}}{\lambda}\, l \left(-\frac{u^2}{\cancel{2}c^2}\right) = 2\,\underbrace{\frac{l}{\lambda}}_{\sim 10^{6}}\underbrace{\left(-\frac{u^2}{c^2}\right)}_{\sim 10^{-8}}$$

Taylor expansion

Formally $f(x) = f(0) + f'(0)\cdot x + \frac{1}{2}f''(0)\,x^2 + \ldots$

Practically - very useful to simplify calculations for small parameters.

Useful expansions

$$e^x = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^n}{n!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n+1}\frac{x^n}{n} + \ldots$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \ldots + \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)}{n!}x^n + \ldots$$

Particularly useful

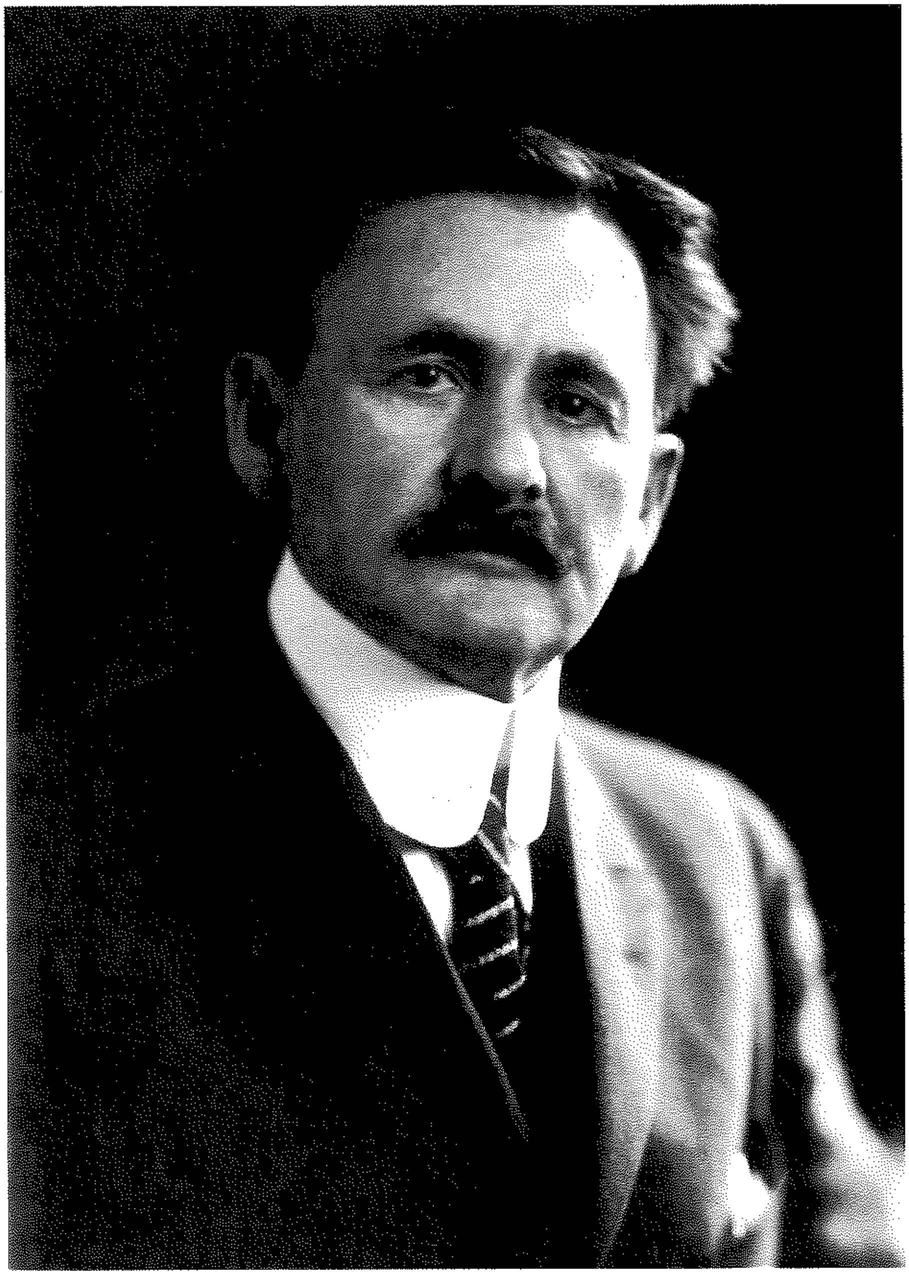
$$\sqrt{1+x} = (1+x)^{1/2} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$$

$$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2} \approx 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \ldots$$

$$\frac{1}{1+x} = (1+x)^{-1} \approx 1 - x + x^2 + \ldots$$

Sometimes handy for simple arithmetic

$$\frac{1}{0.95} = \frac{1}{1-0.05} \approx 1 + 0.05 = 1.05$$



Albert Michelson (1852 - 1931)

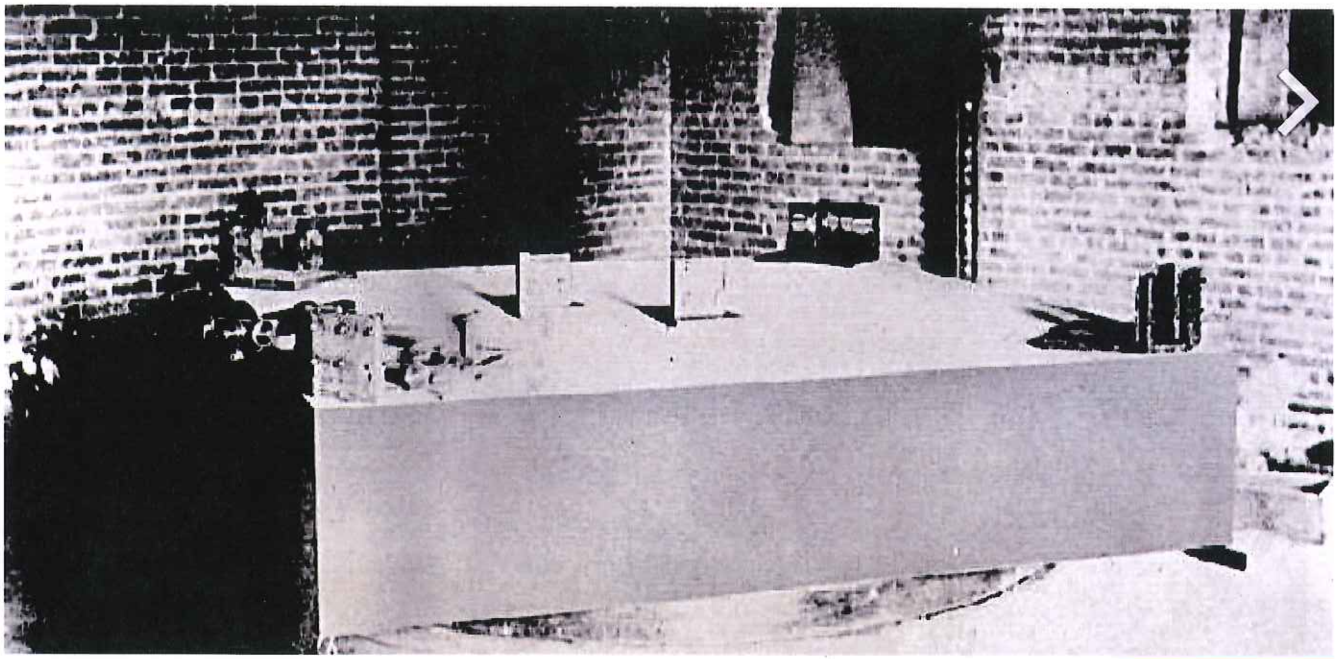


Figure 1. Michelson and Morley's interferometric setup, mounted on

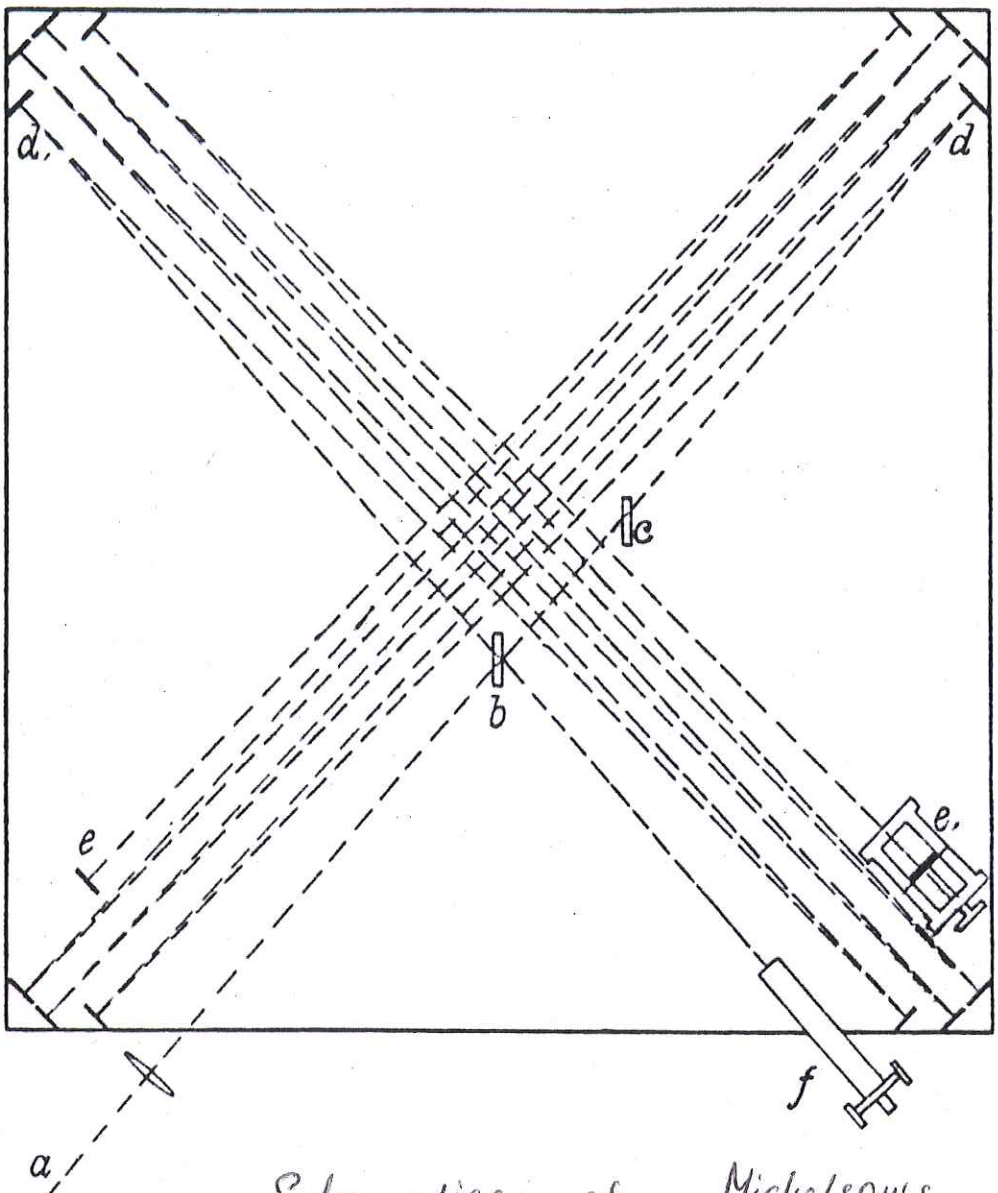
More details

Case Western Reserve University - http://www.cellularuniverse.org
/AA2MM_Aether.htm

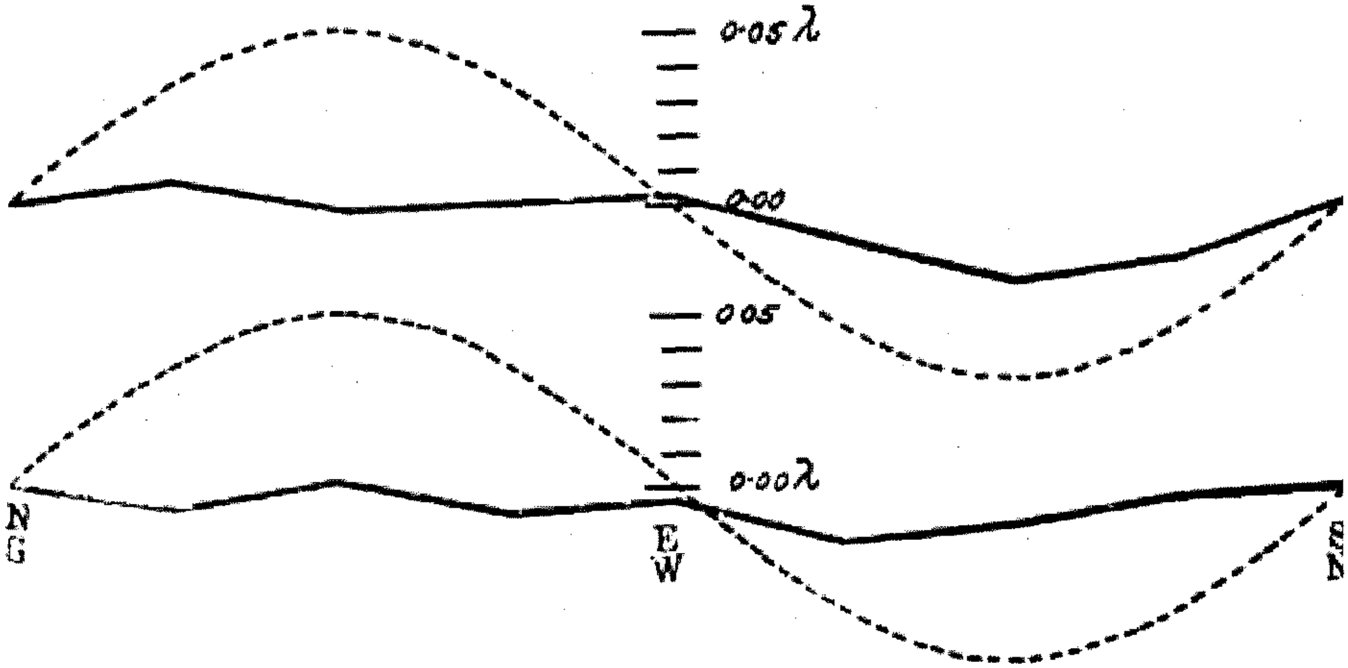
The Setup of Michelson Morley experiment in 1887 at what is now Case Western Reserve University.

Public Domain
File: Michelson morley experiment 1887.jpg
Created: circa 1887

4.



Schematics of Michelson's experimental setup



Dashed lines - theory prediction assuming that the Earth rotates inside the stationary aether; [reduced by the factor of 8]

Solid lines - experimental observations