

**Relevant equations you may or may not need:**

Lorentz transformations 
$$\begin{cases} x' = \gamma(x - vt) \\ y' = y; \quad z' = z \\ t' = \gamma\left(t - \frac{v}{c^2}x\right) \end{cases} \quad \begin{cases} u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \\ u'_{y,z} = \frac{u_{y,z}}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} \end{cases} \quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Length contraction  $L = L_0/\gamma$ ; time dilation  $\tau = \gamma\tau_0$ ,

Doppler effect:  $f_{\text{obs}} = f_{\text{source}} \sqrt{\frac{1 + v/c}{1 - v/c}}$  (approaching observer)

Relativistic energy and momentum:  $\vec{p} = \gamma m \vec{v}$ ;  $E^2 = p^2 c^2 + m^2 c^4$ ;  $\frac{\vec{v}}{c} = \frac{\vec{p}c}{E}$   
 $E = \gamma m c^2$

Photoelectric effect:  $hf = \phi + K_{\text{max}}$       Optical transitions:  $hf = E_{\text{ini}} - E_{\text{fin}}$

Wave-particle duality  $E = \hbar\omega = hf$ ,  $p = \hbar k = \frac{h}{\lambda}$ ;  $k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi f}{c}$

Uncertainty principle  $\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$ ;  $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

General Schrödinger equation:  $i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t)$

$P_{a < x < b} = \int_a^b |\Psi(x,t)|^2 dx$ ; Normalization:  $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$ ;

Time-independent 1D Schrödinger equation:  $-\frac{\hbar^2}{2m} \frac{d^2 \psi_n(x)}{dx^2} + U(x)\psi_n(x) = E_n \psi_n(x)$ ;  $\Psi_n(x,t) = \psi_n(x) e^{-iE_n t/\hbar}$

1-D infinite square well  $0 < x < L$ :  $E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$ ;  $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$ .

Simple harmonic oscillator  $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$

General solutions for the equation  $\frac{d^2 \psi}{dx^2} + k^2 \psi = 0$  are  $A \cos(kx) + B \sin(kx)$  or  $C e^{ikx} + D e^{-ikx}$

General solution for the equation  $\frac{d^2 \psi}{dx^2} - \alpha^2 \psi = 0$  is  $A e^{\alpha x} + B e^{-\alpha x}$

Boundary conditions: a wave function is (a) always continuous; (b) smooth (derivative is continuous) unless  $U(x) = \infty$ .

Hydrogen-like ion energy levels:  $E_n = -\frac{Z^2 m_e (ke^2)^2}{2\hbar^2 n^2} = -\frac{Z^2 E_R}{n^2}$

Angular momentum values:  $L^2 = \hbar^2 l(l+1)$ ;  $l = 0(s), 1(p), 2(d), 3(f), \dots, n-1$   
 $L_z = \hbar m$        $m = -l, -l+1, \dots, l-1, l$

Spin values:  $S^2 = \hbar^2 s(s+1)$ ;  $S_z = \hbar m_s$ ; for electron, proton, neutron:  $s = \frac{1}{2}$

$E_{\text{rot-vib}} = \frac{\hbar^2}{2I} \ell(\ell+1) + \left(\nu + \frac{1}{2}\right) \hbar\omega$ ;

Nuclear binding energy  $E_b = (Z \cdot m_p + N \cdot m_n - M_A) c^2$

Nuclear nomenclature  ${}^A_Z X$

*Some useful constants*

$c = 3 \cdot 10^8$  m/s

$h = 2\pi\hbar = 6.63 \cdot 10^{-34}$  J·s

$h = 4.14 \cdot 10^{-15}$  eV·s

$hc = 1240$  eV·nm

$\hbar c = 197$  eV·nm

$m_e = 0.51$  MeV/c<sup>2</sup>

$m_p = 938.3$  MeV/c<sup>2</sup>

$m_n = 939.6$  MeV/c<sup>2</sup>

$u = 931.5$  MeV/c<sup>2</sup>

$E_R = \frac{m_e (ke^2)^2}{2\hbar^2} = 13.6$  eV

*Handy trig identities*

$e^{ix} = \cos x + i \sin x$

$e^{-ix} = \cos x - i \sin x$

$\cos x = (e^{ix} + e^{-ix})/2$

$\sin x = (e^{ix} - e^{-ix})/2i$

$|e^{ix}|^2 = 1$