Physics 102H Midterm test #3
April 12 2024

Name (please print): Solutions

This test is administered under the rules and regulations of the honor system of the College of William & Mary.

Signature: ______________________________

Final score: ___________

Some useful constants

$k = 1.38 \times 10^{-23}$ J/K $N_A = 6.022 \times 10^{23}$ $R = kN_A = 8.315$ J/mol · K 0°C=273K

one atmosphere = 760 mm Hg = 10^5 Pa 1 cal = 4.186 J 1amu = 1.66×10^{-27}kg

$e=1.6\cdot10^{-19}$ C $\varepsilon_0=8.84\cdot10^{-12}$ C^2/Nm^2 $k=9\cdot10^9$ Nm^2/C^2 $k = \frac{1}{4\pi\varepsilon_0}$

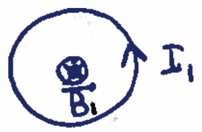
$\mu_0=4\pi\cdot10^{-7}$ H/m

Show all work to receive credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.

Problem 1 (35 points)

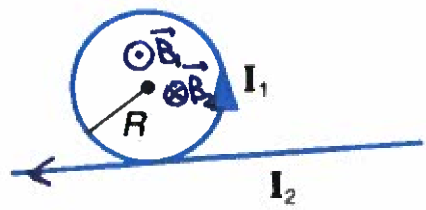
a) A circular loop of wire with radius R=0.25m carries constant current electric current I_1=1.2A counterclockwise. What is the direction and magnitude of the magnetic field in the center of the loop?

$$B = \frac{\mu_0}{4\pi}\frac{2\pi R I}{R^2} = \frac{\mu_0 I}{2R} = 3.02\cdot 10^{-6}\,T$$



out of the page

b) A long straight wire is placed tangentially to the loop. Find the direction and the magnitude of the electric current in the wire I_2 required to null the magnetic field in the center of the loop.



$\vec{B}_2$ (from the wire) – into the page

$B_2 = B_1$

$$\frac{\mu_0 I_1}{2R} = \frac{\mu_0 I_2}{2\pi R} \Rightarrow I_2 = \pi \cdot I_1 = 3.76A$$

to the left

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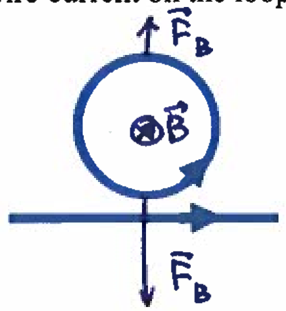
c) If the straight wire and the loop are in the same plane, indicate possible effects of the straight wire current on the loop. Circle the correct choice and, when appropriate, provide the direction.



i) There is a net force on the loop: No

(Yes) (indicate the direction)

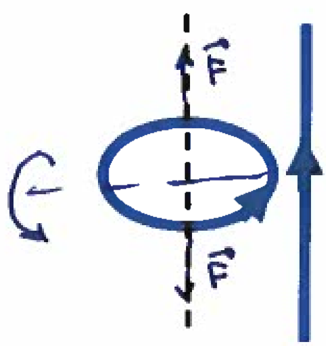
ii) There is a net torque on the loop **(No)**

Yes (indicate the rotation direction)

i) Since magnetic field is stronger closer to the wire the net force is toward the wire (down)

ii) One of possible explanations: $\vec{\mu}$ is out of the page and parallel to $\vec{B}$, so $\vec{\tau} = \vec{\mu} \times \vec{B}$ is zero

d) If the straight wire is perpendicular to the plane of the loop, indicate possible effects of the straight wire current on the loop. Circle the correct choice and, when appropriate, provide the direction.



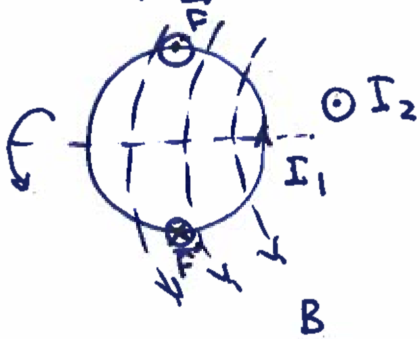
i) There is a net force on the loop: **(No)** Because of the symmetry

Yes (indicate the direction) all forces cancel out

ii) There is a net torque on the loop: No

(Yes) (indicate the rotation direction)

Top view



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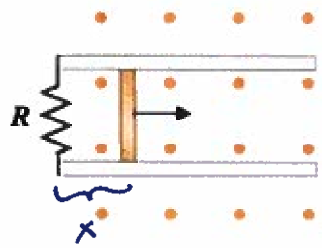
Problem 2 (25 points)



The figure shows a top view of a bar that can slide without friction. The resistor is $R = 6.0\ \Omega$, and a magnetic field $B = 3.20$ T is directed out of the page and fills the whole area. The length of the bar is $l = 1.20$ m.

(a) Calculate the applied force required to move the bar to the right at a constant speed $v = 3.3$ m/s.

$\Phi = B\ell \cdot x$

$\frac{d\Phi}{dt} = B\ell \frac{dx}{dt} = B\ell v$

$\mathcal{E} = B\ell v = 12.67\,V$

$I = \mathcal{E}/R = \frac{B\ell v}{R} = 2.11\,A$

$F = I\ell B = \frac{B^2\ell^2 v}{R} = 8.11\,N$

(b) What is the direction of the induced current?

Since the flux is increasing, the generated current will create magnetic field opposing to existing one. Thus it flows ~~counter~~ clockwise

(c) At what rate is the energy dissipated at the resistor? Show that this value matches the power required to move the bar with the constant speed.

Dissipated energy rate (power) $P_R = I^2 \cdot R = \frac{B^2\ell^2 v^2}{R}$

Power ~~Work~~ of the applied force $P_F = F \cdot v = \frac{B^2\ell^2 v^2}{R}$

$P = 26.76$ Watts

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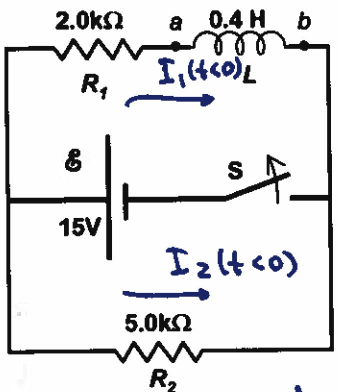
Problem 3 (40 points)

In the electric circuit below, the switch is closed at time $t<0$, so that constant current flows though the battery. The switch is open at $t=0$ (i.e. no current through the battery for $t \geq 0$).

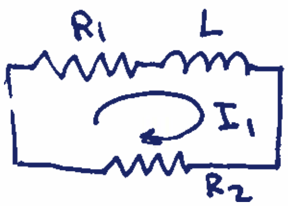


a) What are the steady-state ($t<0$) currents in the resistors R_1 and R_2?

$I_1(t<0) = \mathcal{E}/R_1 = 0.0075A = 7.5mA$

$I_2(t<0) = \mathcal{E}/R_2 = 0.003A = 3mA$

b) What are the current and the voltage drops across the resistors R_1 and R_2 immediately after $t=0$?



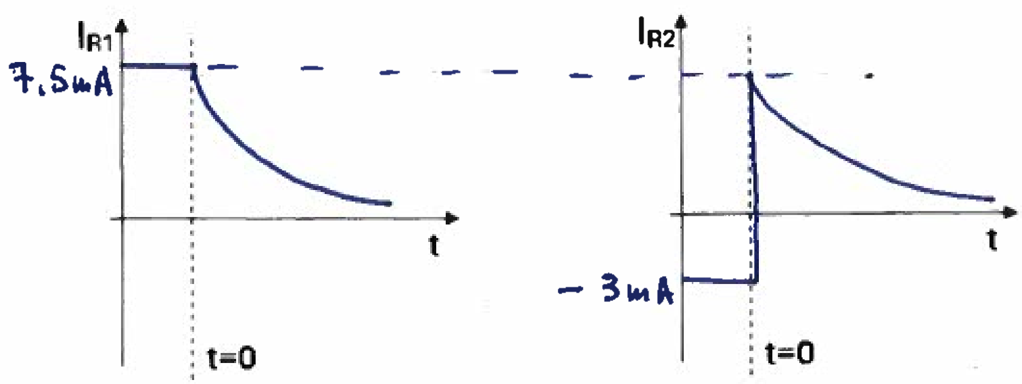
at $t=0+$ the current through L and R_1 does not change: $I_1 = \mathcal{E}/R_1 = 7.5mA$, $V_{R_1} = I_1 R_1 = 15V$

Current through I_2 is the same as $I_1 = 7.5mA$

$V_{R_2} = I_1 \cdot R_2 = 37.5V$

c) Make a free-hand graph of the currents in the resistors R_1 and R_2 as a function of time (including both $t<0$ and $t>0$).

Current through R_2 changes direction



d) At what moment after $t_0>0$ does the current in R_1 have the value I_0=2mA?

LR circuit

$R_{eq} = R_1 + R_2 = 7k\Omega$ $\qquad \tau_{LR} = L/R_{eq} = 5.7 \cdot 10^{-5} s$

$I(t) = I_1 e^{-t/\tau_{LR}}$

$2mA = 7.5mA\, e^{-t/\tau_{LR}}$

$t = \tau_{LR} \ln\left(\frac{7.5mA}{2mA}\right) = 7.55 \cdot 10^{-5} s$

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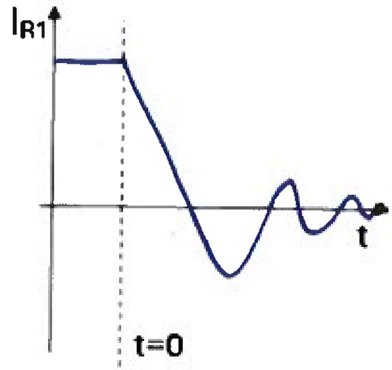
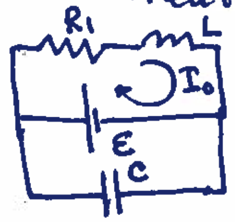
e) Let's imaging that resistor R_2 is replaced by a capacitor C long before the switch is open. Sketch the current in the resistor R_1 as a function of time (including both $t<0$ and $t>0$) and qualitatively explain its behavior.



$t<0$: the capacitor is fully charged, no current through that branch



$I_0 = \mathcal{E}/R_1$

$t>0$: once the switch is closed, current will start flowing, discharging the capacitor, and transferring the stored electrical energy from the capacitor to the inductor and then back → oscillating circuit.

However, since energy is lost at the resistor, the oscillations decay.

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