Problem 1 (25 points)

In fair weather, the electric field in the air immediately above the Earth's surface is E_0= 120N/C directed downward.

a) If we approximate the Earth as a perfectly round conducting sphere, what is the electric charge of the Earth? *Some potentially useful values: radius of the Earth R_E = 6370km, mass of the Earth $M_E = 6\times10^{24}$ kg.*

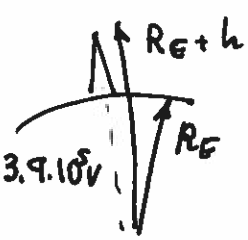
E-fild points down → negative charge

$$E_0 = \frac{kQ}{R_E^2} \Rightarrow Q = E_0 \frac{R_E^2}{k} = 4\pi\varepsilon_0 E_0 R_E^2$$

$$Q = -5.4\cdot10^5 C$$

b) What is the difference in electric potential between the top and the bottom of the Eiffel tower, h=324m tall?

$$V(r) = \frac{kQ}{r}$$



$$\Delta V = \frac{kQ}{R_E} - \frac{kQ}{R_E+h} = \frac{kQ\cdot h}{R_E(R_E+h)} \approx \frac{kQ\cdot h}{R_E^2} = E_0\cdot h = 3.9\cdot10^4 V$$

Basically, if you're close to a surface of a giant sphere, the electric field is almost constant.

c) An extravagant French millionaire decided to compensate the Earth's electric field directly underneath the Eiffel tower. To achieve that he pays to place a point charge on the top of the tower. What should be the sign and the magnitude of this charge to ensure that the total electric field immediately above the ground is zero? *Neglect all the effects of metal structure of the tower itself and its urban surroundings.*

$$E_{comp} = E_0$$

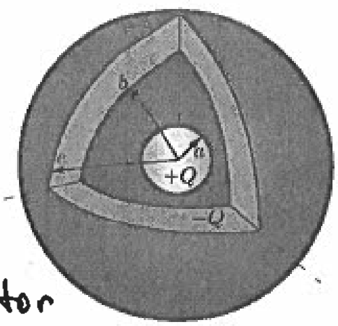
$$\frac{kq}{h^2} = E_0 \qquad q = E_0 \frac{h^2}{k} = 4\pi\varepsilon_0 E_0 h^2 = 1.4 mC$$

Compensation field must point up, toward the charge → negative charge.

~~[illegible]~~ $q = -1.4\cdot10^{-3} C$

Show all work to receive credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.

Problem 2 (35 points)



A spherical non-conducting shell has an inner radius b, an outer radius c, and carries a total charge $-Q$ that is distributed uniformly. Located in the cavity of and concentric with the shell is a smaller spherical conductor with radius a and total charge $+Q$.

a) In what area (or areas) the electric field is zero?

$r<a$ E-field is zero inside a conductor

$r>c$ $Q_{enc}=0$

b) Calculate electric field values in the remaining regions.

$a<r<b$ $E(r)=\frac{kQ}{r^2}$ (no influence of the shell)

$b<r<c$ Charge density inside the shell

$$\rho = {-Q}\Big/{\tfrac{4\pi}{3}(c^3-b^3)}$$

For a Gaussian surface sphere of a radius r

$$Q_{enc} = Q+\rho\cdot\frac{4\pi}{3}(r^3-b^3) = Q-Q\,\frac{r^3-b^3}{c^3-b^3} = Q\,\frac{c^3-r^3}{c^3-b^3}$$

$$E(r) = \frac{kQ_{enc}}{r^2} = \frac{kQ}{r^2}\,\frac{c^3-r^3}{c^3-b^3}.$$

c) How much work should be done to move a small test charge q_{test} from the surface of the conducting sphere to the inner wall of the shell?

~~$W = kQ\,q_{test}$ for a~~

$$W = U(r=b)-U(r=a)$$

Inside the shell the potential due to the shell is constant, so doesn't contribute to the work

$$W = q_{test}\left[V_{cond}(b)-V_{cond}(a)\right] = kQ\,q_{test}\left[\frac{1}{b}-\frac{1}{a}\right]$$

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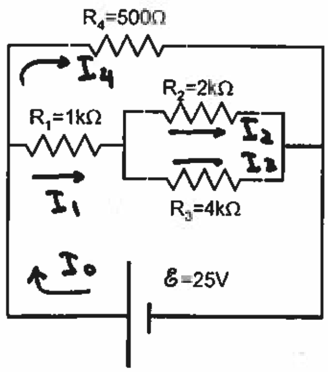
Problem 3 (40 points) $1k\Omega = 1000\Omega$

a) In the circuit below calculate total current drawn from the battery. Show the direction of the current in the diagram.



Equivalent resistance calculations:

$$R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \frac{4}{3}\,k\Omega = 1333\,\Omega$$

$$R_{123} = R_1 + R_{23} = \frac{7}{3}\,k\Omega = 2333\,\Omega$$

$$R_{tot} = \left(\frac{1}{R_{123}} + \frac{1}{R_4}\right)^{-1} = \frac{7}{17}\,k\Omega = \frac{7000}{17}\,\Omega = 411\,\Omega$$

$$I = \frac{\mathcal{E}}{R_{tot}} = 0.061\,A = 61\,mA \qquad \boxed{1mA = 10^{-3}A}$$

b) What is the voltage drop across the resistor R_1?

$$I_1 = \mathcal{E}/R_{123} = 25V/2333\Omega = 0.011\,A = 11\,mA$$

$$V_{R_1} = I_1 \cdot R_1 = 11V \quad (\text{or } 10.7V \text{ to be precise})$$

c) What is the current through the resistor R_2?

$$V_{R_2} = V_{R_3} = \mathcal{E} - V_{R_1} = 25V - 10.7V = 14.3V$$

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{14.3V}{2000\Omega} = 0.007A = 7mA$$

c)Which resistor conducts the highest current? Which one conducts the minimum current? You don't need to calculate current values, but you need to explain your answers.

R_4 must have the highest current, since the full battery voltage is applied across the smallest resistor.
R_3 must have the lowest current, since the current through R_1 splits, and lower current flows throu a brunch with highest resistance

d) If more than P_{max}=2W of power is dissipated in the resistor R_4 it overheats and starts smoking. What is the maximum voltage the battery may have to avoid this calamity?

$$P = V^2/R_4 \qquad V_{max} = \sqrt{P_{max} \cdot R_4} = \sqrt{2W \cdot 500\Omega} = 31.6\,V$$

Show all work to receive credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.