Physics 102H Midterm test #1
February 16 2024

Name (please print): solutions

This test is administered under the rules and regulations of the honor system of the College of William & Mary.

Signature: ______________________

Final score: __________

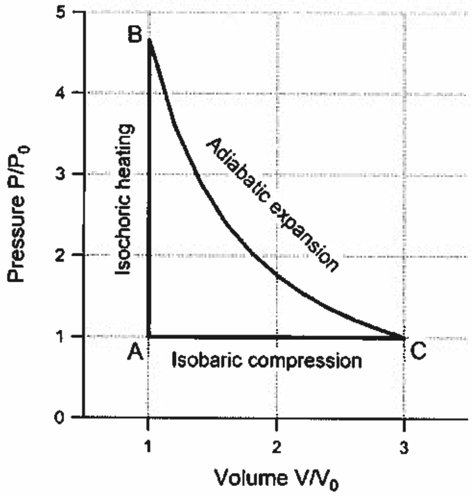
Some useful constants

$k = 1.38 \times 10^{-23}$ J/K $N_A = 6.022 \times 10^{23}$ $R = kN_A = 8.315$ J/mol · K 0°C=273K

one atmosphere = 760 mm Hg = 10^5 Pa 1 cal = 4.186 J 1amu = 1.66×10^{-27}kg

Show all work to receive full credit, and circle your final answers. This exam is closed book, and you can use calculators only for simple arithmetical operations.

Problem 1 (40 points)



The Lenoir cycle is an idealized thermodynamic cycle often used to model a pulse-jet engine. The cycle consists of three stages: first, the gas is heated at constant volume, then it undergoes adiabatic expansion, and finally the exhaust at constant pressure. Shown below is an example of Lenoir cycle in which one mole of an ideal diatomic gas ($\gamma = C_P/C_V = 7/5$) is taken though the cycle. In the starting point A the volume of the gas is V_0, and its pressure is P_0.

a) What is the lowest and the highest temperature T_L and T_H the gas experiences during the cycle?

A: $P_0 V_0 = nRT_A = RT_A \Rightarrow T_A = \frac{P_0 V_0}{R}$ ← lowest

$n = 1$

C: $3P_0 V_0 = nRT_C \Rightarrow T_C = \frac{3P_0 V_0}{R}$

B: $P_B V_B^{\gamma} = P_C V_C^{\gamma} \Rightarrow P_B \cdot V_0^{\gamma} = P_0 (3V_0)^{\gamma} \Rightarrow P_B = 3^{\gamma} P_0 = 4.66 P_0$

$T_B = \frac{P_B \cdot V_0}{R} = 4.66 \frac{P_0 V_0}{R}$ ← highest

b) What is the highest achievable efficiency for any heat engine operating between such high and low temperature reservoirs?

$$e_{max} = e_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{1}{3^{\gamma}} = 78.5\%$$

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c) What is the efficiency of the Lenoir cycle? How does it compare to the maximum achievable efficiency?

$$e = \frac{W}{Q_H} = \frac{Q_H - |Q_L|}{Q_H}$$

$$Q_H = Q_{AB} = C_V (T_B - T_A) = \frac{5}{2}R(3^{\gamma}-1)\frac{P_0V_0}{R} = \frac{5}{2}(3^{\gamma}-1)P_0V_0 = 9.14P_0V_0$$

$$Q_L = Q_{CA} = C_p(T_A - T_C) = \frac{7}{2}R\left(-2\frac{P_0V_0}{R}\right) = -7P_0V_0$$

$$e = \frac{Q_H - |Q_L|}{Q_H} = 1 - \frac{|Q_L|}{Q_H} = 1 - \frac{7P_0V_0}{9.14P_0V_0} = 23.4\%$$

d) If one desires to operate a <u>heat pump</u> on this particular Lenoir cycle, what changes (if any) need to be made? What would be the coefficient of performance for such a heat pump?

The cycle must be run in reverse

$$COP = \frac{|Q_H|}{W} = \frac{|Q_H|}{|Q_H| - Q_L} = 4.27 \text{ or } 427\%$$

Problem 2 (30 points)

A physics student decides to send her friend a nice balloon to cheer him up after a harsh physics midterm. She chooses a *3.0*-liter (1 liter = 10^{-3} m^3) balloon filled with ^{4}He atoms, each with mass $m_{He}=6.6\times\cdot 10^{-27}kg$.

a) If an average rms speed of a He atom is $v_{rms} = 1350$ m/s, what is the temperature of the gas?

$$V_{RMS} = \sqrt{\frac{3kT}{m}} \Rightarrow T = \frac{m V_{RMS}^2}{3k} = \frac{6.6\cdot 10^{-27} kg \cdot (1350)^2 m^2/s^2}{3\cdot 1.38\cdot 10^{-23} J/K}$$

$$T = 290.5K \quad \text{or} \quad 17.5°C$$

b) At this temperature how many He atoms must be inside the balloon to maintain it at atmospheric pressure?

$$P\cdot V = N\cdot k_B T \qquad N = \frac{P\cdot V}{k_B\cdot T} = 7.5\cdot 10^{22} \text{ atoms}$$

c) How cold is it outside if the balloon's radius shrinks by 3% when outdoors?

$$P = \text{const} \qquad \frac{V_{in}}{T_{in}} = \frac{V_{out}}{T_{out}}$$

$$T_{out} = T_{in}\left(\frac{V_{out}}{V_{in}}\right) = T_{in}\left(\frac{r_{out}}{r_{in}}\right)^3 = (0.97)^3 T_{in}$$

$$T_{out} = 0.912\cdot T_{in} = 265K \quad \text{or} \quad -7.8°C$$

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Problem 3 (30 points)

(a) What is the minimal height from which one needs to drop a *10-g* ice cube at *0°C* so that it completely evaporates at impact?

Assume that the temperature of the cube does not change during the fall, and that all the energy generated by the impact goes into heating the cube.

Some possibly useful properties of water: boiling point is 100°C, melting point is 0°C, latent heat of vaporization $L_V = 2.26\times10^3$ *J/kg, specific heat c= 4186 J/kg·K, latent heat of fusion* $L_F = 3.34\times10^5$ *J/kg.*

$$mgh_{min} = L_f \cdot m + mc\Delta T + L_v \cdot m \qquad \Delta T = 100\,K$$

$$h_{min} = \frac{1}{g}(L_f + c\Delta T + L_v) = 77026\,m$$

(b) How much does the entropy of ice change from the moment of impact to its evaporation? It may be helpful to separately calculate the change for each phase of its transformation.

Melting: $\Delta S_{melting} = \frac{Q}{T_{melt}} = \frac{mL_F}{273K} = 12.23\ J/K$

Evaporating: $\Delta S_{vap} = \frac{mL_v}{T_{vap}} = \frac{mL_v}{373K} = 0.061\ J/K$

Heating: $\Delta S_{heating} = mc\ln\frac{T_{vap}}{T_{melt}} = mc\ln\frac{373K}{273K} = 13.06\ J/K$

$\Delta S_{total} = 25.4\ J/K$

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