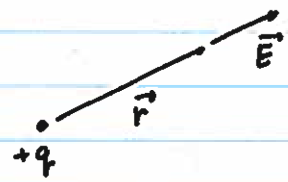
Electric potential and potential difference



Electric field of a point charge

$$\vec{E}(\vec{r}) = \frac{kq}{r^2}\hat{r}$$

Electric potential

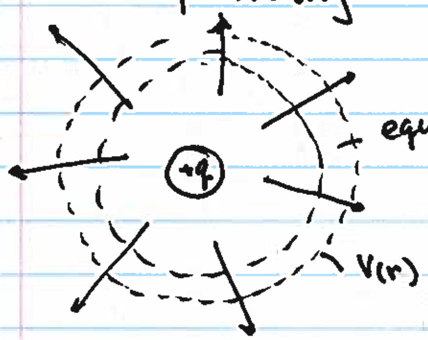
$$V(\vec{r}) = \frac{kq}{r}$$

Many charges: $\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \dots$ vector sum

$V_{tot} = V_1 + V_2 + \dots$ scalar sum

In general $\vec{E} = \left\{ -\frac{\partial V}{dx}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right\}$

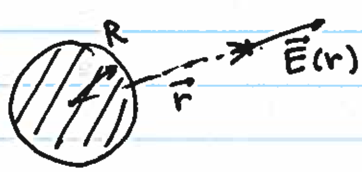
Spherically symmetric potential



$$E_r = -\frac{dV(r)}{dr}$$

In these cases when we used Gauss law to calculate electric field, we can then use it

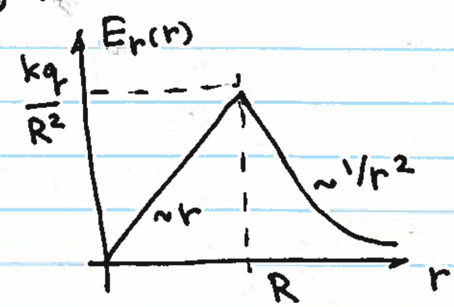
Solid charged sphere



$$\vec{E}(r) = \begin{cases} \frac{kq}{r^2}\hat{r} & r > R \\ \frac{kq}{R^3}\vec{r} & r < R \end{cases}$$

if $E_r(r) = -\frac{dV}{dr}$ $\quad V(r) = -\int E_r(r)dr + \text{const}$

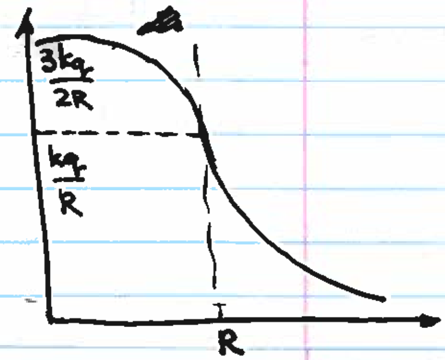
$$V(r) = \begin{cases} \frac{kq}{r} & r > R \\ -\frac{kq}{2R^3}r^2 + \text{const} & r < R \end{cases}$$



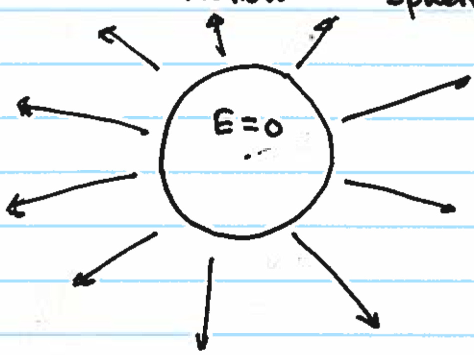
Since potential V represent energy, it must be continuous, so

at $r = R$ $\quad \frac{kq}{R} = -\frac{kq}{2R} + \text{const} \Rightarrow \text{const} = \frac{3kq}{2R}$

$$V(r) = \begin{cases} \frac{kq}{r} & r > R \\ -\frac{kq}{2R^3} r^2 + \frac{3kq}{2R} & r < R \end{cases}$$



Hollow sphere



Outside $\quad V(r) = \frac{kq}{r} \quad r > R$

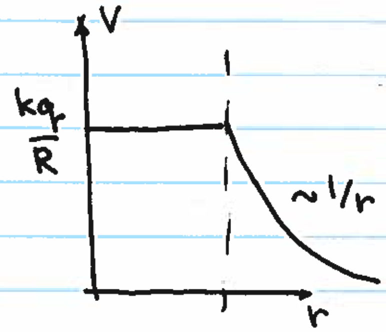
Inside: $\quad E = 0 \Rightarrow V = \text{const}$

to be continuous

$V(R) = kq/R$

so anywhere inside $V = \frac{kq}{R}$

$$V(r) = \begin{cases} \frac{kq}{r} & r > R \\ \frac{kq}{R} & r < R \end{cases}$$



That makes sense: if E inside is zero, it takes no work to move charges around, so their electrostatic potential energy is constant.

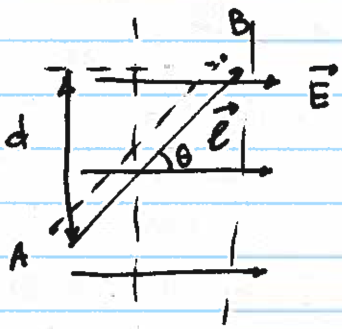
Electrostatic potential is defined up to a constant (basically, we deside where it is zero)

More physically useful thing — potential difference (voltage). It represents the amount of work done when moving a test charge from point A to point B

$$W_A = W_B + [\text{work}] \Rightarrow V_A = V_B + \frac{\text{work}}{q_{test}}$$

$$\underline{\Delta V_{AB} = V_A - V_B}$$

Constant electric field



$$W_{on} = +\vec{F}\cdot\vec{\ell} = q_{test}\cdot\vec{E}\cdot\vec{\ell}$$

$$V_A - V_B = \frac{\text{work}}{q_{test}} = \vec{E}\cdot\vec{\ell} =$$

$$= E \cdot \ell \cos\theta = E \cdot d$$

equipotential plane

Conductors — electric field inside must be zero, since otherwise free charges would move freely until it is zero.

Moreover, the surface of a conductor must have the same potential value, so that there is no force making charges move along the surface. We can have force / electric field working perpendicular to the surface, since charges cannot leave the surface.