

# Optics - the science of light

What is light?  $\vec{E}(x,t) = \vec{E}_0 \cos(kz - \omega t)$

Classical description: electromagnetic wave  
frequency - always the same in any medium  $\omega$  [rad/s],  $\nu$  [Hz]  
 $\nu = \omega / 2\pi$

$$c = 3 \cdot 10^8 \text{ m/s}$$

speed of light  
in vacuum

wavelength -  $\lambda_0 = \frac{2\pi c}{\omega} = \frac{c}{\nu}$  in vacuum  
 $k_0 = 2\pi / \lambda_0$

Wavelength changes when light propagates  
in the transparent material

Speed of light in material  $v = c/n$   
 $n$  - refractive index

$$\lambda = \frac{2\pi v}{\omega} = \frac{2\pi c}{n\omega} = \frac{\lambda_0}{n} \quad k = \frac{2\pi n}{\lambda_0} = \frac{2\pi}{\lambda}$$

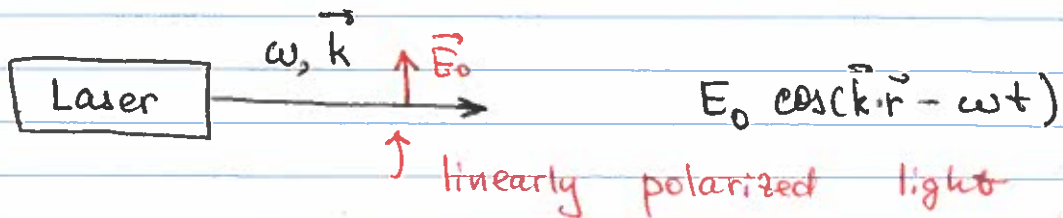
Because vast majority of materials  
respond much stronger to electric component  
of EM wave, we typically use  $\vec{E}$  parameters  
to characterize light

$$\text{Intensity} = \frac{\text{energy}}{\text{area} \cdot \text{time}} = \frac{E_0^2}{2\mu_0 c}$$

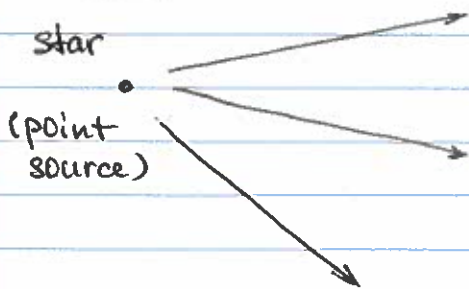
Direction of  $\vec{E}$  - polarization of light

Plain EM wave gives a decent approximation for a ray of monochromatic (single frequency) light.

In homogenous medium or in vacuum light travels in a straight line.



Most natural sources of light are not monochromatic



light is emitted in all directions, but then travels straight (in an empty space)

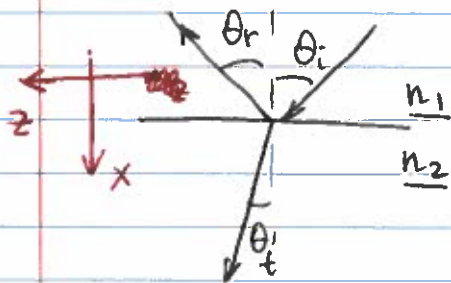
In this case the intensity of light will drop with distance as  $1/r^2$  (same energy is spread over ~~the~~ the area  $4\pi r^2$ )

Also, most natural sources contain many frequencies (spectrum)

What happens when the light hits the boundary?

1. Reflection  
(reflected beam goes back into the same medium)

2. Transmission  
(transmitted beam penetrates into the second medium)



Reflection law:  $\theta_i = \theta_r$

Refraction law (Snell's law)

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

Where these laws come from?

$$\vec{E}_{inc} = \vec{E}_{o,inc} \cos(k_{ix} \cdot x + \underline{k_{iz} \cdot z} - \omega t)$$

$$\vec{E}_{ref} = \vec{E}_{o,ref} \cos(k_{rx} \cdot x + \underline{k_{rz} \cdot z} - \omega t)$$

$$\vec{E}_{tr} = \vec{E}_{o,tr} \cos(k_{tx} \cdot x + \underline{k_{tz} \cdot z} - \omega t)$$

must be the same at the surface  $x=0$  for any  $z$  and  $t$

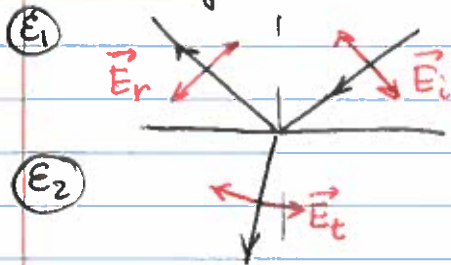
$$k_{iz} = k_{rz} \Rightarrow \frac{\omega \cdot n_1}{c} \sin \theta_i = \frac{\omega \cdot n_1}{c} \sin \theta_r$$

$$\boxed{\theta_i = \theta_r}$$

$$k_{iz} = k_{tz} \Rightarrow \frac{\omega \cdot n_1}{c} \sin \theta_i = \frac{\omega \cdot n_2}{c} \sin \theta_t$$

$$\boxed{n_1 \sin \theta_i = n_2 \sin \theta_t}$$

If we need to know how much energy is reflected and transmitted, we'll have to look more closely on Maxwell's equations



Perpendicular

~~Parallel~~ to the surface

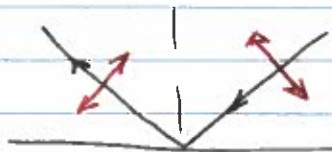
$$\epsilon_2 E_{2\perp} - \epsilon_1 E_{1\perp} = \delta$$

(surface charge density)

Parallel to the surface

$$\vec{E}_{2\parallel} = \vec{E}_{1\parallel}$$

Reflection off the metal



All energy is reflected (perfect mirror)

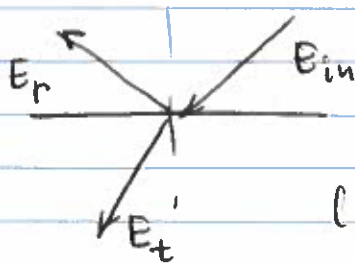
$$E_2 = 0$$

Induced surface charge

↓  
surface charge wave called surface plasmons

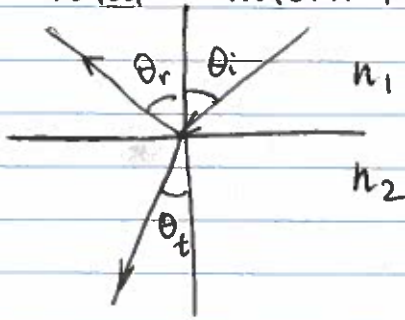
Used to enhance electric field at the surface

Transmission off transparent boundary



reflection and transmission coefficients depend on light polarization (in plane of the boundary or not) and the incidence angle.

## Total internal reflection



$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

air  $\rightarrow$  glass

$$n_1 < n_2 \rightarrow \theta_i > \theta_t$$

glass  $\rightarrow$  air

$$n_1 > n_2 \rightarrow \theta_i < \theta_t$$

$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i \leq 1$$

## Critical incidence angle

$$\frac{n_1}{n_2} \sin \theta_{cr} = 1$$

$$\sin \theta_{cr} = n_2/n_1$$

