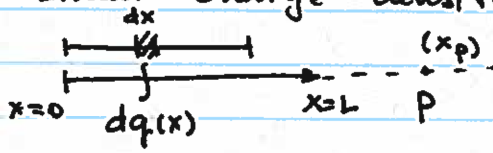
Continuous charge distribution

1D Linear charge density



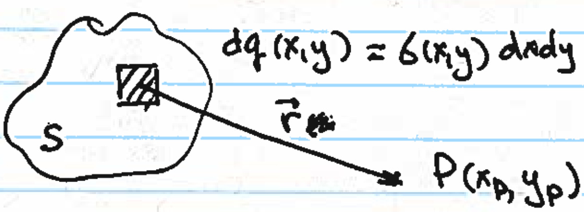
$$\lambda(x) = \frac{dq(x)}{dx}$$

$$E(x_p) = \int_0^L dE(x) = \int_0^L \frac{k\,dq(x)}{(x_p-x)^2} = \int_0^L \frac{k\lambda(x)\,dx}{(x_p-x)^2}$$

$\times\ \mathrm{sgn}(x_p - x)$

$$V(x_p) = \int_0^L \frac{k\,dq}{(x_p-x)} = \int_0^L \frac{k\lambda\,dx}{(x_p-x)}$$

2D Surface charge density



$$dq(x,y) = \sigma(x,y)\,dx\,dy$$

$$\sigma(x,y) = \frac{dq(x,y)}{dx\,dy}$$

$$\vec{E}(x_p, y_p) = \iint_S d\vec{E}(x,y) = \iint_S \frac{k\,dq(x,y)}{r^2}\,\hat{r}$$

where $\vec{r} = [(x_p - x), (y_p - y)]$

$$= \iint_S dx\,dy \frac{k\,\sigma(x,y)}{[(x_p-x)^2 + (y_p-y)^2]^{3/2}} \{(x_p-x), (y_p-y)\}$$

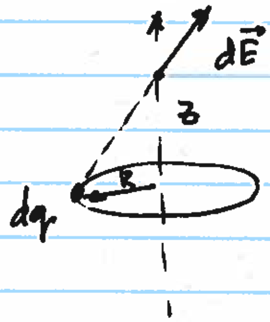
$$V(x_p, y_p) = \iint_S \frac{k\,dq(x,y)}{r} = \iint \frac{k\,\sigma\,dx\,dy}{\sqrt{(x_p-x)^2 + (y_p-y)^2}}$$

3D Volume charge density

$$dq(x,y,z) = \rho(x,y,z)\,dx\,dy\,dz$$

$$\rho(x,y,z) = \frac{dq(x,y,z)}{dx\,dy\,dz}$$

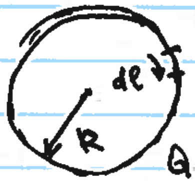
Uniformally - charged ring



(Detailed treatment in previous notes)

Need only $dE_z = \dfrac{kz\,dq}{(z^2+R^2)^{3/2}}$

Top view



$dq = \lambda\, d\ell$

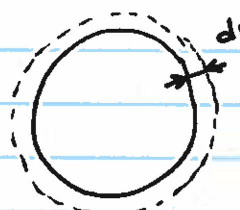
Q_{ring}

$$E_z(z) = \int dE_z = \int \frac{kz\,dq}{(z^2+R^2)^{3/2}} =$$

$$= \frac{kz}{(z^2+R^2)^{3/2}} \int dq = \frac{kz\,Q_{ring}}{(z^2+R^2)^{3/2}}$$

$$dV = \frac{k\,dq}{\sqrt{z^2+R^2}} \Rightarrow V(z) = \frac{kQ_{ring}}{\sqrt{z^2+R^2}}$$

Uniformly charged <u>disc</u>



$$dq_{ring} = \sigma \cdot 2\pi r\, dr$$

$$dE_{z\,ring} = \frac{k\,z}{(z^2+r^2)^{3/2}}\, dq_{ring} =$$

$$\sigma = \frac{Q_{disc}}{\pi R^2} \qquad = \frac{2\pi k \sigma r z\, dr}{(z^2+r^2)^{3/2}}$$

$$E_{z\,disc} = \int_0^R \frac{[2\pi k \sigma z]\, r\,dr}{(z^2+r^2)^{3/2}} =$$

$$= 2\pi k \sigma z \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}} = 2\pi k \sigma z \left[-\frac{1}{\sqrt{z^2+r^2}} \right]\Bigg|_0^R =$$

$$= 2\pi \sigma k z \left(\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}} \right) = 2\pi \sigma k \left[1 - \frac{z}{\sqrt{z^2+R^2}} \right]$$

$R \to \infty$ infinite plane $E_{plane} = 2\pi \sigma \cdot k$

Electric potential of a disc

$$dV = \frac{k\,dq}{\sqrt{z^2+r^2}} = \frac{k \cdot 2\pi r \sigma\, dr}{\sqrt{z^2+r^2}}$$

$$V = \int_0^R \frac{2\pi k \sigma \quad r\,dr}{\sqrt{z^2+r^2}} = 2\pi k \sigma \sqrt{z^2+R^2}\,\Big|_0^R =$$

$$= 2\pi k \sigma \left(\sqrt{z^2+R^2} - |z|\right)$$

Here it is a little trickier to jump to the expression for the infinite plane since $V \to \infty$ as $R \to \infty$

Here we have to remember that only <u>difference</u> in potential energy matters. So we can set $V=0$ at any convenient location

$$V = 2\pi k \sigma \left(\sqrt{z^2+R^2} - |z|\right) \overset{R \gg z}{\simeq} 2\pi k \sigma (R - |z|) + V_0$$

Lets set $V=0$ at $z=0$ $\Rightarrow V_0 = -2\pi k \sigma R$

Then $V = -2\pi k \sigma \cdot |z| = -\dfrac{\sigma}{2\varepsilon_0} \cdot z = -E_z \cdot |z|$