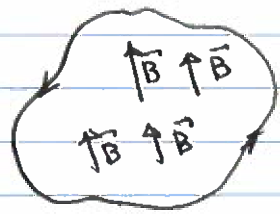
Electro-magnetic induction

The remaining Maxwell's equation

$$\oint_{loop} \vec{E}\, d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} = -\frac{\partial}{\partial t}\int \vec{B}\, d\vec{A} = -\frac{\partial}{\partial t}\Phi_{\vec{B}}$$

$\vec{B}$ – field flux



$\oint \vec{E}\, d\vec{l}$ = voltage drop around the loop = EMF $\mathcal{E}$

$$\mathcal{E} = -\frac{d\Phi_{\vec{B}}}{\partial t}$$

<u>Faraday's law of induction</u>

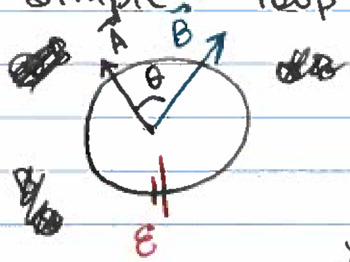
An electric current is induced in a loop by changing (in time) magnetic field

or more precisely

An induced emf is produced in a loop by the changing magnetic field flux (through this loop)

We assume that the magnetic field is uniform <u>in space</u>

Simple loop



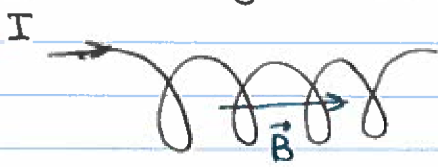
$$\Phi_B = \int \vec{B}\, d\vec{A} = B \cdot A \cdot \cos\theta$$

If the flux is changing, it induces current in the loop, just like a "fictional" battery

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B \cdot A \cdot \cos\theta)$$

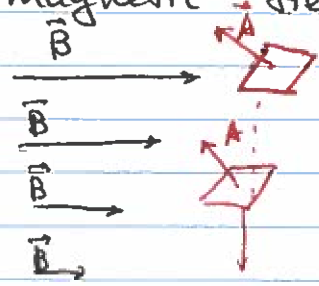
How can we change magnetic flux?

1. Change value of the magnetic field



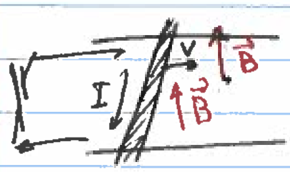
If a current through the solenoid changes, $\vec{B}$ is changing → EMF is induced in the coil (self-inductance)

2. Move the loop through inhomogeneous magnetic field



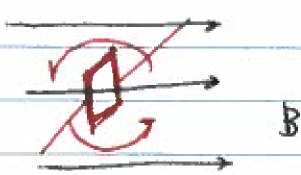
Moving a loop from stronger to weaker field induce current in its (eddie currents)

3. Change the area of the loop



Motional emf

4. Change the angle b/w the loop normal and the magnetic field.



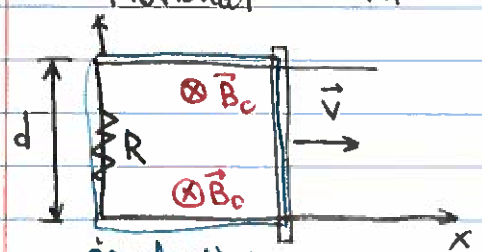
Electric turbine – rotating loop in constant magnetic field

How to figure out the direction of the induced current / emf?

Lenz's law: the current induced in a loop is in the direction that creates a magnetic field opposing the change of flux through the loop

Nature will try to keep the flux unchanged! ~~Indu~~ (Nature is conservative!) Induced current tends to maintain the original flux through the area.

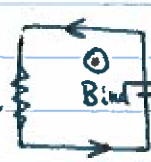
Motional EMF



Magnetic flux

$$\Phi_B = B_0 \cdot A = B_0 \cdot d \cdot x$$

$$\frac{d\Phi_B}{dt} = B_0 d \frac{dx}{dt} = B_0 d v$$



$\mathcal{E} = B_0 \cdot d \cdot v$

Which way current runs?

- Φ_B grows since the loop area increases
  - ↳ Smaller B will keep flux the same
    - ↳ induced current tries to decrease the total magnetic field
      - ↳ induced B_{ind} is in the opposite direction than $\vec{B}_0$

Induced current $I = \frac{\mathcal{E}}{R} = \frac{B_0 \cdot d \cdot v}{R}$