Written assignment #9 solutions

Q2 a) The differential distance light travels b/w two arms is

$$2(L_1+\Delta x) - 2L_1 = 2\Delta x$$

To switch from constructive to destructive interference $2\Delta x = \lambda/2 \Rightarrow \Delta x = \lambda/4$

b) The electric field in the first arm after the roundtrip

$$E_1 = E_0\, e^{ik\cdot 2L_1 - i\omega t}$$

and in the second arm

$$E_2 = E_0\, e^{ik2(L_1+\Delta x) - i\omega t}$$

$$E_{tot} = E_1 + E_2 = E_0\, e^{ik\cdot 2L_1 - i\omega t} + E_0\, e^{ik2(L_1+\Delta x) - i\omega t}$$

$$= E_0\, e^{i2kL_1 - i\omega t}\, e^{ik\Delta x} \underbrace{\left[e^{ik\Delta x} + e^{-ik\Delta x}\right]}_{2\cos k\Delta x}$$

Amplitude $2E_0 \cos k\Delta x$ – after recombination

Amplitude before splitting – $2E_0$

$$P_{tr} \propto (2E\cos k\Delta x)^2 = (2E)^2 \cos^2 k\Delta x$$

$$P_{inc} \propto (2E)^2$$

$$P_{tr} = P_0 \cos^2 k\Delta x = P_0 \cos^2 \frac{2\pi\Delta x}{\lambda}$$

c) $\Delta x = 10^{-23} L = 4\cdot 10^{-20}$ m

$$P_{dark} = P_0 \sin^2 \frac{2\pi\Delta x}{\lambda} \approx P_0 \left(\frac{2\pi\Delta x}{\lambda}\right)^2 = 5{,}6\cdot 10^{-25}\ W$$

d) $\dfrac{P_{dark}\cdot 1ms}{2\pi\hbar/\lambda} = 0.95$ photon

Q3 a) $K = e \cdot V = 1.6 \cdot 10^{-19} C \cdot 100V = 1.6 \cdot 10^{-17} J$

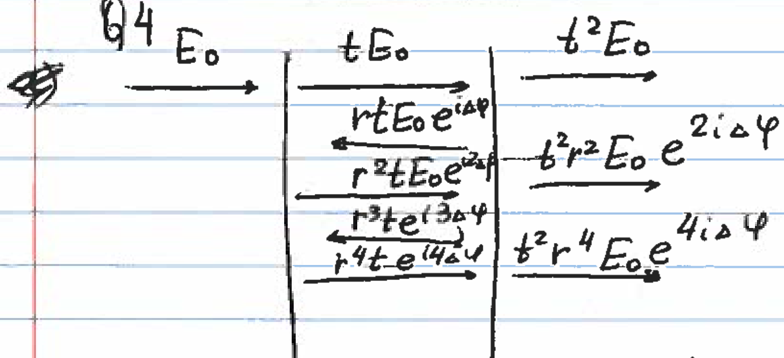
b) $K = \frac{1}{2} m v^2 = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$

$$\lambda_e = \frac{2\pi\hbar}{p} = \frac{2\pi \cdot 10^{-34} J \cdot s}{\sqrt{2m \cdot K}} = \frac{2\pi \cdot 10^{-34}}{\sqrt{2 \cdot 9 \cdot 10^{-31} kg \cdot 1.6 \cdot 10^{-17} J}}$$

$$\lambda_e = 1.23 \cdot 10^{-10} m = 1.23 \text{ Å}$$

c) If the pathlength changes by $\lambda_e/2 = 0.61$ Å, the sign of interference will change.

Q4



$\Delta\varphi = k \cdot L = \frac{2\pi L}{\lambda}$

a) $E_{n+1} = t^2 r^{2n} e^{i2n\Delta\varphi}$ $\quad n = 0, 1, 2 \dots$

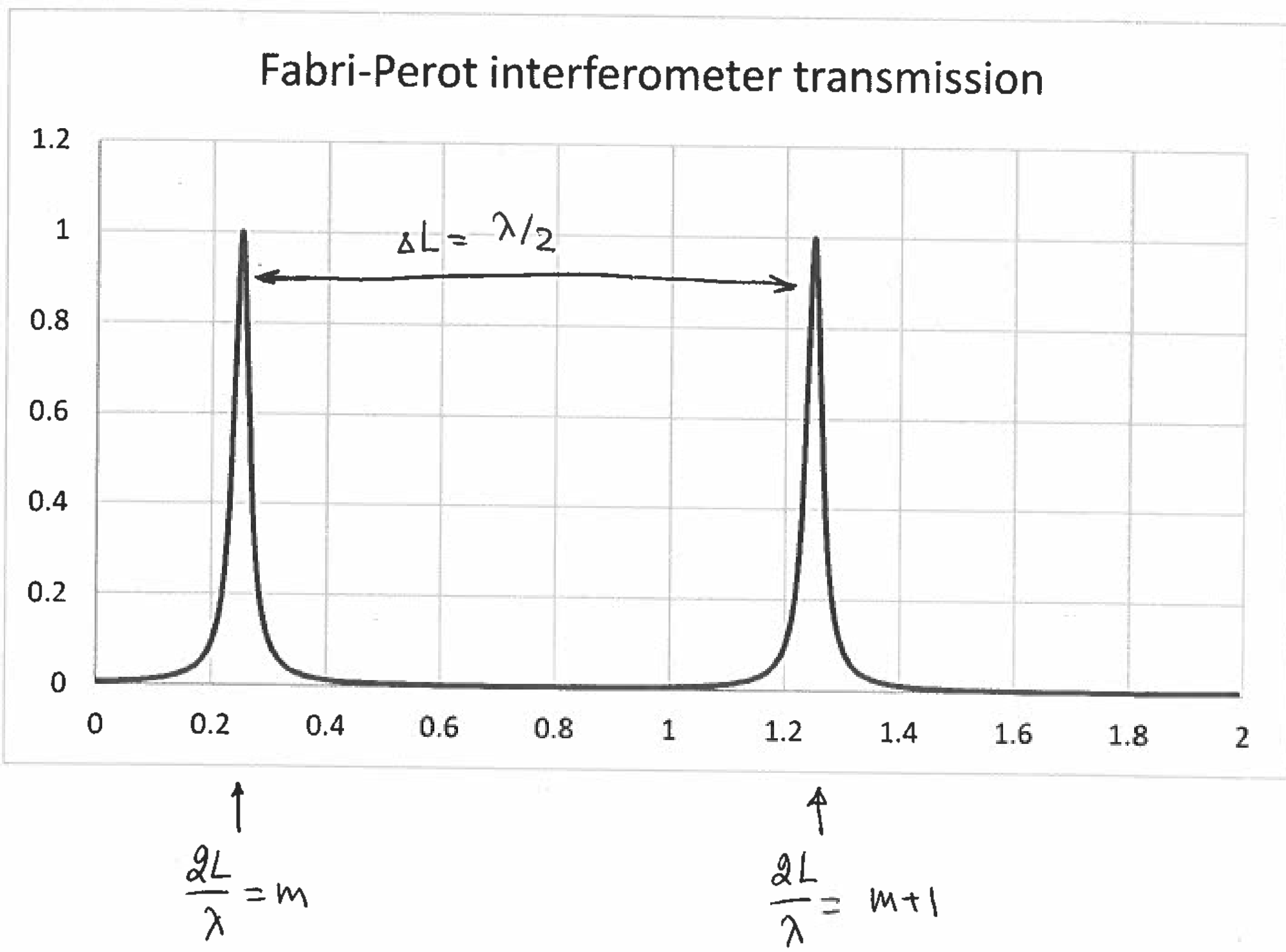
b) $E_{tot} = E_1 + E_2 + E_3 + \dots = t^2E_0 + t^2r^2E_0e^{2i\Delta\varphi} + t^2r^4E_0e^{4i\Delta\varphi} + \dots = t^2E_0 \cdot \sum_{n=0}^{\infty} r^{2n}e^{i2n\Delta\varphi} =$

$$= \frac{t^2E_0}{1 - r^2e^{i2\Delta\varphi}} = \frac{t^2E_0}{1 - r^2 \exp(i\frac{4\pi L}{\lambda})}$$

c) $|E_{tot}|^2 = E_{tot} \cdot E_{tot}^* = \frac{(t^2E_0)^2}{(1-r^2e^{i4\pi L/\lambda})(1-r^2e^{-i4\pi L/\lambda})} =$

$$= \frac{(t^2E_0)^2}{1+r^4-2r^2(e^{i4\pi L/\lambda}+e^{-i4\pi L/\lambda})} = \frac{(t^4E_0)^2}{1+r^4-2r^2\cos(4\pi L/\lambda)}$$

$T = \frac{|E_{tot}|^2}{|E_0|^2} = \frac{(1-r^2)^2}{1+r^4-2r^2\cos(4\pi L/\lambda)}$; $T_{max} = 1$ for $\frac{4\pi L}{\lambda} = 2\pi m$ $\quad m = 0, 1, 2 \dots$



Fabri-Perot interferometer transmission
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$\Delta L = \lambda/2$
$\frac{2L}{\lambda} = m$
$\frac{2L}{\lambda} = m+1$