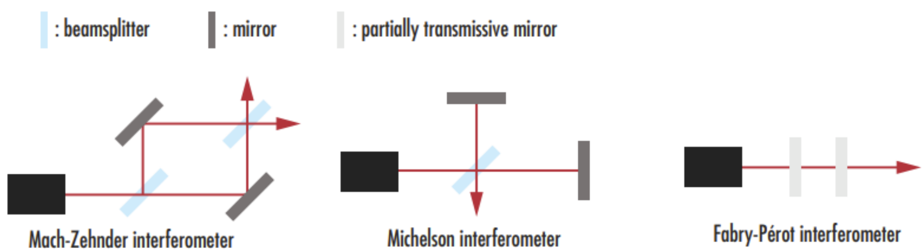
General Physics II Honors (PHYS 102H)

Problem set # 9 *(due April 25)*
All problems are mandatory, unless marked otherwise. Each problem is 10 points.

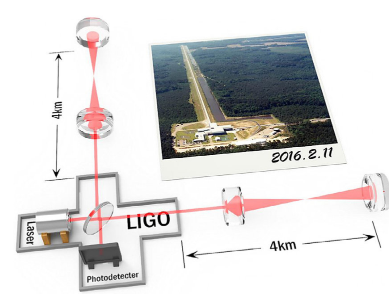
Q1 Use your phone or camera to take a picture that illustrates the wave nature of light (interference, diffraction, etc.) You can use internet for inspirations, but I trust you to submit your own picture (it can be freshly recorded or from your archive). Make sure to explain how this picture illustrates the wave nature of light in 2-3 sentences.

The next three questions explore the concept of interferometers that are currently used a lot of precise measurements. Interferometers typically use a beamsplitter to split light from a single source into a test beam and a reference beam of equal energy. The beams are recombined before reaching a photodetector, and any optical path difference between the two paths create interference. There are several common interferometer configurations.



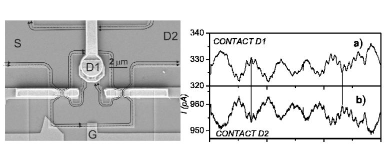
Mach–Zehnder interferometers utilize one beamsplitter to separate an input beam into two separate paths. A second beamsplitter recombines the two paths into two outputs, which are sent to photodetectors. **Michelson interferometers** use a single beamsplitter for splitting and recombining the beams. **Fabry–Perot** interferometers allow for multiple trips of light by using two parallel partially transparent mirrors instead of two separated beam paths.

Q2 Michelson interferometer was originally designed by Michelson to find the aether, and nowadays it forms the basis for the gravitational wave detectors - LIGO and VIRGO.



a) Assuming that the distance between the beam splitter and each of the mirrors is L_1 and $L_2 = L_1 + \Delta x$, by what distance Δx the second mirror should move to shift the output light brightness from maximum to minimum?
b) Using the approach similar to the double-slit interference calculations we did in class, show that the power of the recombined laser beam after the beam-splitter is $P_{tr} = P_0 \cos^2(2\pi\Delta x/\lambda)$. (For an electromagnetic wave power P is the total energy flux, but for these calculations it is enough to just consider that $P \propto \langle |E|^2 \rangle$.)
c) The relative space distortion created by a gravitational wave is $\Delta x/L \approx 10^{-23}$. In LIGO experiment $L = 4$km, and the its laser operates at $\lambda = 1064$nm with the power of $P_0 = 10$W. If we assume that without the wave the interference is completely destructive (no light is transmitted), calculate the expected transmitted light power (in this case $P_{dark} = P_0 \sin^2(2\pi\Delta x/\lambda)$) when the gravitational wave arrives.
d) The energy of a single photon is $2\pi\hbar c/\lambda$. Approximately how many photons will emerge from the "dark" interferometer port, if the gravitational wave lasts for 2 ms. *The answer may surprise you, but read how LIGO boosts its signal).*

Q3 Mach-Zender interferometer design is preferable for interference measurements with single quantum particles, since it is easier to make a measurements of the recombined beams after the second beam splitter. Such an interferometer has been created not only for photons, but also for electrons and atoms. In the sample circuit shown below each electron can simultaneously follow two electric wires, so that after the recombination the measured current displays oscillations. The same rules for constructive and destructive interference apply, but for massive particle one has to calculate the de Broglie wavelength $\lambda_e = 2\pi\hbar/p$, where $\hbar = 10^{-34} J \cdot s$ is the Planks constant, and p is the momentum of the electron.



(a) If an electron was accelerated from rest by applying 100 V voltage, its kinetic energy is said to be 100 eV. How much is that in SI units?
(b) What is the de Broglie wavelength of such an electron?
(c) By how much the pathlength in one arm of the Mac-Zender interferometer must change to change the interference pattern in the same detector from constructive to destructive? *These are relatively slow electrons, so that you can use the expression $mv^2/2$ to calculate their kinetic energy. In Modern physics you'll discover that this expression is not valid for relativistic (fast) particles.*

Q4 In Michelson interferometer the change in output power with a mirror displacement is $cos^2(2\pi\Delta x/\lambda)$ which is a relatively smooth function. Fabri-Perot interferometer allows to create a much sharper transmission peaks, and thus it is ideal for more precise measurements. In the Fabri-Perot interferometer each of two mirrors transmits r fraction of the electromagnetic field amplitude, and transmits t fraction (such that $r^2 + t^2 = 1$). Usually $t \ll 1$, so each mirror individually reflects most of the light. Nevertheless, for certain distances between them two mirrors can transmit almost all the light! We will explore it step by step:
(i) Let's start with an incoming beam of amplitude E_0 before the first mirror. We can Write its amplitude immediately after the first mirror as tE_0.
(ii) When light travels from the first mirror to the second, their spatial phase varies as e^{ikx}, so by the time it hits the second mirror, its electric field becomes tE_0e^{ikL} (here $k = 2\pi/\lambda$ is the light wave vector, and L is the distance b/w the mirrors.
(iii) Now the light is reflected off the second mirror (its field rtE_0e^{ikL}. Most of it travels back to the first mirror, again acquiring the phase shift, so that right before it hits the first mirror again its electric field is rtE_0e^{2ikL}. However, a little part of it leaks through, and its amplitude to the overall output light is $E_1 = t^2E_0e^{ikL}$.
Now this reflection cycle repeats itself infinite number of times, and each time a fraction of light is transmitted through the second mirror (toward our detector).
(a) Write the value of the transmitted electric field E_n after n cycles between the mirrors.
(b) Write an infinite sum representing the total electric field transmitted through the second mirror E_{tr}. Recognize that this is a geometrical progression and write down an exact solution.
(c) Calculate the intensity transmission $T = |E_{tr}|^2/|E_0|^2$. What is its maximum value, and for what values of L it occurs?
(d) Sketch T as the function of L/λ for $r = 0.95$ to show one or few transmission resonances. You can pick any reasonable value of L, since the transmission is a periodic function of L/λ, and thus the exact value of L is not important.