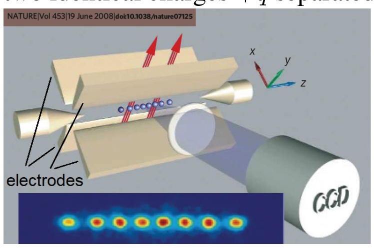
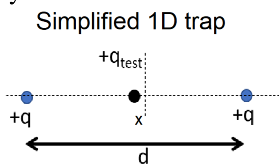
General Physics II Honors (PHYS 102H)

Problem set # 4 *(due March 1)*
All problems are mandatory, unless marked otherwise. Each problem is 10 points.

Q1 Trapped ions are one of the leading candidates for realization of quantum computers. For building quantum gates it is important to keep ions localized in a so-called ion trap. One of most common ion traps is Poul trap, in which ions are placed between four mutually perpendicular electrodes that create zero electric field in a center. Then individual ion qubits form a long chain in the center of the trap.

Here we will consider a simplified 1D case, where ions only can move in one direction. Such 1D trap can be realized using two identical charges $+q$ separated by a distance d.

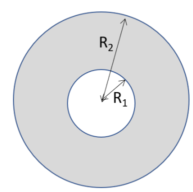




(a) Write down a net force acting on a test charge q_{test} placed between these two charges, and show that it is zero in the middle.
(b) Assuming that the test charge moves by a small displacement $x \ll d$ away from the trap center, use Taylor expansion to show that the resulting force is restorative and obeys the Hooke's law $F = -k_{Hooke}x$. *Hint:* The Taylor expansion you may find useful is $\frac{1}{(d/2 \pm x)^2} \approx \frac{1}{(d/2)^2}\left(1 \mp \frac{4x}{d}\right)$.
(c) Since it is impossible to cool ions completely, they actually going to oscillate around the equilibrium position, just like a mass on a spring. If the trap charges are $q = 10e$, the test charge $q_{test} = +e$, and the distance $d = 1$ mm, calculate the trap frequency using the spring constant k_{Hooke} from part (b). In practice, measuring this frequency is often used to characterize the trapping field in an experimental ion traps. Electron mass is $m_e = 9.1 \cdot 10^{-31}$ kg and $e = 1.6 \cdot 10^{-19}$C.

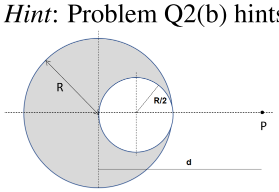
Q2 A total charge Q is distributed uniformly throughout a spherical shell of inner and outer radii R_1 and R_2, as shown.
(a) Show that the electric field E as a function of the distance from the shell's center r is:

$$\vec{E}(r) = \begin{cases} 0 & r < R_1 \\ \frac{Q}{4\pi\epsilon_0 r^2}\frac{r^3 - R_1^3}{R_2^3 - R_1^3}\hat{r} & R_1 \le r \le R_2 \\ \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} & r > R_2 \end{cases}$$



(b) Show that the same result can be obtained for the region $R_1 \le r \le R_2$ by presenting a spherical shell as a superposition of a positively charged solid sphere of radius R_2 and a smaller sphere of radius R_1 negatively charged with the same value of charge density.

Q3 Consider a uniformly charged shell with charge density ρ consisting of a solid sphere of radius R with an extrusion of radius $R/2$ shifted by $R/2$ away from its center, as shown. Calculate the electric field in point P that is at the distance d from the center of the larger sphere.
Hint: Problem Q2(b) hints on how to approach this problem.



Q4 In classical physics we describe a hydrogen atom having a positively-charged proton in the center, and a negatively charged electron moving around. However, according to quantum mechanics, because of the uncertainly principle an electron bound to a proton does not have a well-defined trajectory, but rather have to be described by a probability distribution of finding it in different points in space. Let's use a known quantum mechanical probability function to construct an average electron density $\rho(r') = \frac{(-e)}{\pi a_0^3}e^{-r'/a_0}$, so that the electron charge enclosed in a sphere of radius r is $q_{enc} = \int_0^r \rho(r')4\pi r'^2 dr'$. Now, assuming that the proton is a point $+e$ charge, and electron cloud is distributed around it with the given negative charge density, calculate the total electric field of a hydrogen atom as a function of distance from the proton r. *Hint*: feel free to use WolframAlpha or some other integral solver to calculate the integral - or do it analytically, if you like integration by parts!