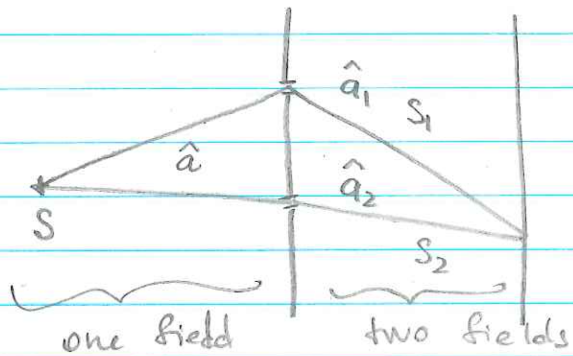
Two-slit interference



$$\hat{E}^{(+)} = K(r) \left[\hat{a}_1 e^{iks_1} + \hat{a}_2 e^{iks_2} \right] e^{-i\omega t}$$

$$f(r) = i \left[\frac{\hbar\omega}{2\varepsilon_0 (2\pi R)} \right]^{1/2} \frac{1}{r} \qquad (s_1 \approx s_2 \approx r)$$

$$I(\bar{r},t) = \mathrm{Tr}\left[\rho E^-(\bar{r},t) E^+(\bar{r},t) \right] = |K(r)|^2 \Big(\mathrm{Tr}(\rho \hat{a}_1^+ \hat{a}) +$$

$$+ \mathrm{Tr}(\rho \hat{a}_2^+ \hat{a}_2) + 2 \left| \mathrm{Tr}(\rho\, \hat{a}_1^+ \hat{a}_2) \right| \cos(k\Delta s + \psi)$$

where ψ is the phase of $\mathrm{Tr}(\rho\, \hat{a}_1^+ \hat{a}_2)$

However, we now need to connect the two modes, emerging from two pin holes with the properties of the single mode of the source S.

$a \qquad b \qquad a_1 + a_2$

one mode + vacuum = two modes

beam - splitter!

$$\hat{a} = \frac{1}{\sqrt{2}} (\hat{a}_1 + \hat{a}_2)$$

$$\hat{b} = \frac{1}{\sqrt{2}} (\hat{a}_1 - \hat{a}_2) \quad \text{- fictitious mode (no real photons)}$$

Karen Ficenic

In the source region

$$|n\rangle_a |0\rangle_b = \frac{1}{\sqrt{n!}} \hat{a}^{+n} |0\rangle_a |0\rangle_b \Rightarrow$$

$$\Rightarrow \frac{1}{\sqrt{n!}} \frac{1}{(\sqrt{2})^n} (\hat{a}_1^+ + \hat{a}_2^+)^n |0\rangle_1 |0\rangle_2$$

$n = 1$

$$|n\rangle_a |0\rangle_b \Rightarrow \frac{1}{\sqrt{2}} (|1\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2) = |\Psi_1\rangle$$

$$\langle \Psi_1 | \hat{a}_1^+ \hat{a}_1 | \Psi_1 \rangle = \frac{1}{2} \langle 10 | \hat{a}_1^+ \hat{a}_1 | 10 \rangle = \frac{1}{2} = \langle \Psi_1 | \hat{a}_2^+ \hat{a}_2 | \Psi \rangle$$

$$\langle \Psi_1 | \hat{a}_1^+ \hat{a}_2 | \Psi_1 \rangle = \frac{1}{2} \langle 1 0 | \hat{a}_1^+ \hat{a}_2 | 0 1 \rangle = \frac{1}{2}$$

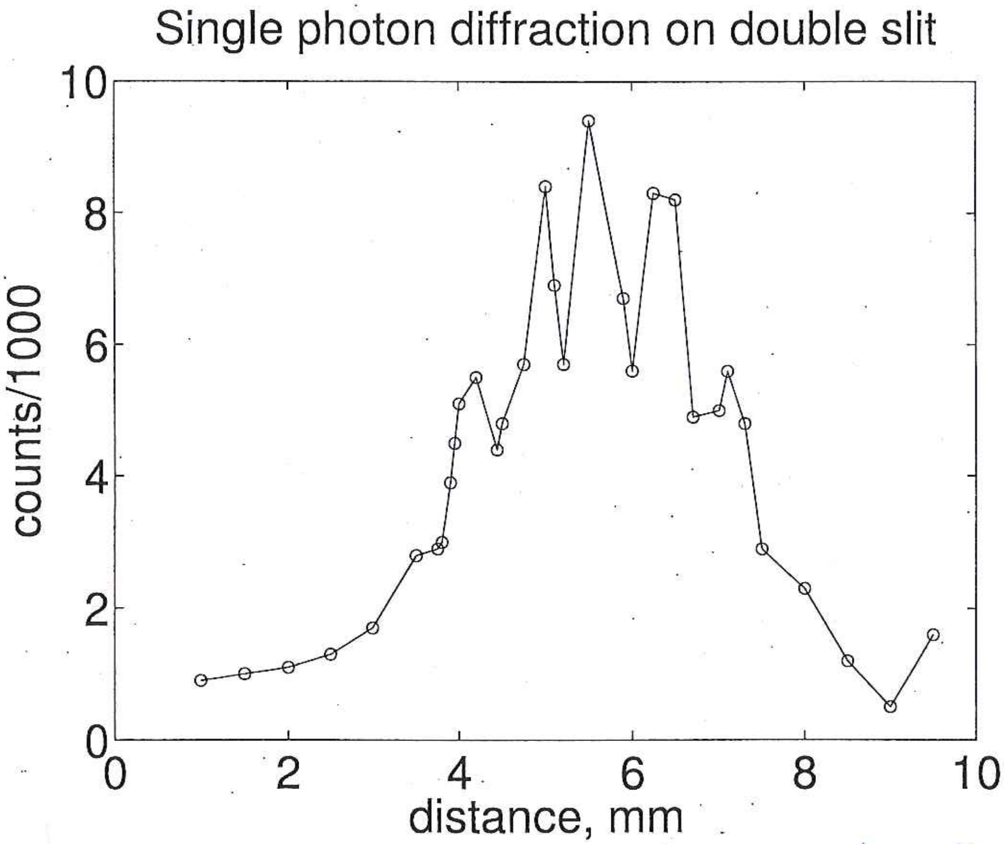
$$I(\vec{r}, t) = |K(\vec{r})|^2 (1 + \cos\varphi)$$

$|\gamma^{(1)}| = 1$ complete coherence

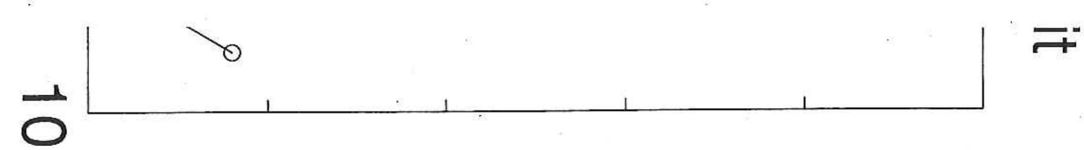
Similarly, for $|n\rangle_a |0\rangle_b \Rightarrow I(\vec{r}, t) = n |K(r)|^2 (1 + \cos\varphi)$

Coherent state! $|\alpha\rangle_a |0\rangle_b \Rightarrow I(\vec{r}, t) = |\alpha|^2 |K(r)|^2 (1 + \cos\varphi)$

Same behavior as for classical states!



Recorded by Karen Ficenic '17
10/3/2015



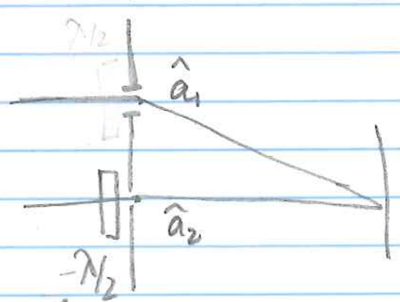
$$\hat{E}^+ = K(r)\left[\vec{e}_x \hat{a}_{1x} e^{iks_1} + \vec{e}_y \hat{a}_{1y} e^{iks_1} + \vec{e}_x \hat{a}_{2x} e^{iks_2} + \vec{e}_y \hat{a}_{2y} e^{iks_2}\right]$$

$$I(\bar{r},t) = |K(r)|^2 \left[\langle \hat{a}^+_{1x} a_{1x}\rangle + \langle \hat{a}^+_{1y} a_{1y}\rangle + \langle \hat{a}^+_{2x} a_{2x}\rangle + \langle a^+_{2y} a_{2y}\rangle\right] +$$

$$+ \left[2|\langle \hat{a}^+_{1x} a_{2x}\rangle|\cos(k\Delta s + \psi_x) + 2|\langle \hat{a}_{1y} a_{2y}\rangle|\cos(k\Delta s + \psi_y)\right]$$

same polarization $\longrightarrow$ full coherence, 100% visibility of the interference picture

What if the polarizations of two photons were altered to be orthogonal before the slits?



Now we can in principle obtain which-way information

$$\tilde{\psi}_1 = \frac{1}{\sqrt{2}}\left(\left|\begin{smallmatrix}1\\0\end{smallmatrix}\right\rangle_1 \left|\begin{smallmatrix}0\\0\end{smallmatrix}\right\rangle_2 + \left|\begin{smallmatrix}0\\0\end{smallmatrix}\right\rangle_1 \left|\begin{smallmatrix}0\\1\end{smallmatrix}\right\rangle_2\right)$$

$$\langle \tilde{\psi}_1 | \hat{a}^+_{1x} \hat{a}_{2x} | \tilde{\psi}_1\rangle = \frac{1}{2}\left\langle\begin{smallmatrix}1\\0\end{smallmatrix}\right|_1 \left\langle\begin{smallmatrix}0\\0\end{smallmatrix}\right|_2 \hat{a}^+_{1x} \hat{a}_{2x} \left|\begin{smallmatrix}0\\0\end{smallmatrix}\right\rangle_1 \left|\begin{smallmatrix}0\\1\end{smallmatrix}\right\rangle_2 =$$

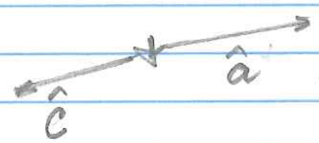
$$= \left\langle\begin{smallmatrix}1\\0\end{smallmatrix}\right|_1 \hat{a}^+_{1x} \left|\begin{smallmatrix}0\\0\end{smallmatrix}\right\rangle_1 \cdot \left\langle\begin{smallmatrix}0\\0\end{smallmatrix}\right|_2 \hat{a}_{2x} \left|\begin{smallmatrix}0\\1\end{smallmatrix}\right\rangle_2 = 0.$$

As expected, which-way information erases interference.

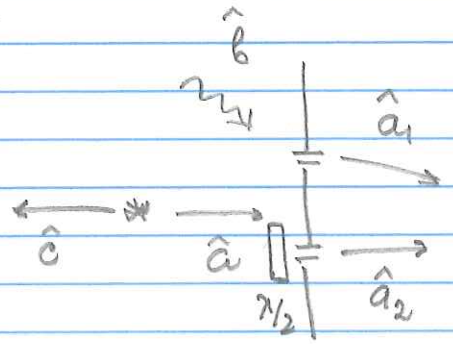
Quantum eraser

Is there a way to erase which-way information? Yes – using entangled states!

$$|\psi_{ac}\rangle = \frac{1}{\sqrt{2}}\left(\left|\begin{matrix}1\\0\end{matrix}\right\rangle_a \left|\begin{matrix}1\\0\end{matrix}\right\rangle_c + \left|\begin{matrix}0\\1\end{matrix}\right\rangle_a \left|\begin{matrix}0\\1\end{matrix}\right\rangle_c\right)$$



If only $\hat{a}$ is measured, its polarisation is not defined; however, we can always use $\hat{c}$ as polarization marker



$\hat{b}$ again is fictitious vacuum mode

Before the slit

$$|\psi_{abc}\rangle = \frac{1}{\sqrt{2}}\left(\left|\begin{matrix}1\\0\end{matrix}\right\rangle_a \left|\begin{matrix}1\\0\end{matrix}\right\rangle_c \left|\begin{matrix}0\\0\end{matrix}\right\rangle_b + \left|\begin{matrix}0\\1\end{matrix}\right\rangle_a \left|\begin{matrix}0\\1\end{matrix}\right\rangle_c \left|\begin{matrix}0\\0\end{matrix}\right\rangle_b\right)$$

After the slits

$$|\tilde{\psi}_1\rangle = \frac{1}{2}\left[\left(\left|\begin{matrix}1\\0\end{matrix}\right\rangle_1 \left|\begin{matrix}0\\0\end{matrix}\right\rangle_2 + \left|\begin{matrix}0\\0\end{matrix}\right\rangle_1 \left|\begin{matrix}1\\0\end{matrix}\right\rangle_2\right)\left|\begin{matrix}1\\0\end{matrix}\right\rangle_c + \left(\left|\begin{matrix}0\\1\end{matrix}\right\rangle_1 \left|\begin{matrix}0\\0\end{matrix}\right\rangle_2 + \left|\begin{matrix}0\\0\end{matrix}\right\rangle_1 \left|\begin{matrix}0\\1\end{matrix}\right\rangle_2\right) \times \left|\begin{matrix}0\\1\end{matrix}\right\rangle_c\right]$$

Which-way information is recorded: the photons passing through two slits are guaranteed to have perpendicular polarizations.
If we measure polarization of $\hat{c}$ to be either $\left|\begin{matrix}1\\0\end{matrix}\right\rangle$ or $\left|\begin{matrix}0\\1\end{matrix}\right\rangle$, we'll collapse the wave function to either the first or the second half of $|\tilde{\psi}_1\rangle$, and returning to the previous case.

Imagine now, however, that we set a polarizer in the channel $\hat{c}$ to be at $45°$ with the two polarizations of the main field.

Thus, we are going to detect a click on the main detector only when there is a $\hat{c}$-photon with polarization $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = |\Psi_{filter}\rangle$

This new state is equally non-orthogonal with the original states $|a_1\rangle, |a_2\rangle$.

$$_c\left\langle \Psi_{filter} \middle| \begin{matrix}1\\0\end{matrix} \right\rangle_c = \left\langle \Psi_{filter} \middle| \begin{matrix}0\\1\end{matrix} \right\rangle_c = \frac{1}{\sqrt{2}}$$

Thus

$$\Psi_{a_1 a_2} = {}_c\langle \Psi_{filter} | \tilde{\tilde{\Psi}}_1 \rangle = \frac{1}{2\sqrt{2}} \left(\left|\begin{matrix}1\\0\end{matrix}\right\rangle_1 \left|\begin{matrix}0\\0\end{matrix}\right\rangle_2 + \left|\begin{matrix}0\\0\end{matrix}\right\rangle_1 \left|\begin{matrix}0\\1\end{matrix}\right\rangle_2 + \left|\begin{matrix}0\\1\end{matrix}\right\rangle_1 \left|\begin{matrix}0\\0\end{matrix}\right\rangle_2 + \left|\begin{matrix}0\\0\end{matrix}\right\rangle_1 \left|\begin{matrix}1\\0\end{matrix}\right\rangle_2 \right)$$

and thus $\langle \Psi_{a_1,a_2} | \hat{a}^{+}_{1x} a_{2x} | \Psi_{a_1,a_2} \rangle =$

$$= \frac{1}{8} \left(\left\langle \begin{matrix}1\\0\end{matrix} \right|_1 \hat{a}^{+}_{1x} \left|\begin{matrix}0\\0\end{matrix}\right\rangle_1 \cdot \left\langle \begin{matrix}0\\0\end{matrix} \right|_2 \hat{a}_{2x} \left|\begin{matrix}1\\0\end{matrix}\right\rangle_2 \right) = \frac{1}{8}$$

Interference is restored! However, only in conditional measurements.

Interestingly enough, one can show that the polarization of $\hat{c}$ can be measured later than the "interference" photon is detected, and just do the post-selection later $\rightarrow$ then the interference appears again. Clearly, somehow the intention of erasing the information is enough to restore the interference.