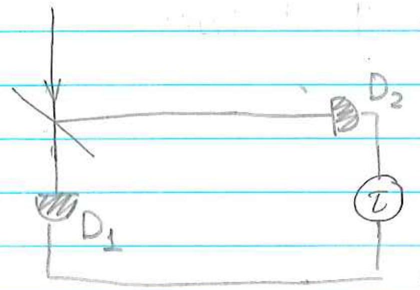
Higher-order coherence function

$\gamma^{(1)} \rightarrow$ interference measure
(spatial / temporal coherence length)
does not depend on light statistics
Measures field amplitude correlations

Hanbury Brown and Twiss experiment
intensity correlations



coincidence count

Coincidence rate $C(\tau) = \langle I(t)\, I(t+\tau) \rangle$

Second-order correlation function = probability of obtaining the coincidence

$$\gamma^{(2)}(\tau) = \frac{\langle I(t)\, I(t+\tau) \rangle}{\langle I(t) \rangle^2} = \frac{\langle E^*(t) E(t) E^*(t+\tau) E(t+\tau) \rangle}{\langle E^*(t)\, E(t) \rangle^2}$$

More general case

$$\gamma^{(2)}(x_1, x_2; x_2, x_1) = \frac{\langle I(x_1) I(x_2) \rangle}{\langle I(x_1) \rangle \langle I(x_2) \rangle} = \frac{\langle E^*(x_1) E^*(x_2) E(x_2) E(x_1) \rangle}{\langle |E(x_1)|^2 \rangle \langle |E(x_2)|^2 \rangle}$$

Clearly $\gamma^{(2)}(\tau) = \gamma^{(2)}(-\tau)$

Monochromatic light $\quad E = E_0 e^{i\omega t}$

$$\langle E^*(t)E^*(t+\tau)\,E(t+\tau)E(t)\rangle = E_0^{\,4}$$

$\gamma^{(2)}(\tau) = 1$ — coherent light

What are possible values of $\gamma^{(2)}(0) = \dfrac{\langle I^2\rangle}{\langle I\rangle^2}$

$$\langle I^2\rangle = \frac{1}{N}\sum_{n=0}^{N} I^2(t_n)$$

$$\langle I\rangle^2 = \left(\frac{1}{N}\sum_{n=0}^{N} I(t_n)\right)^2 = \frac{1}{N^2}\sum_{n,m=0}^{\infty} I(t_n)\,I(t_m)$$

$$2I(t_n)I(t_m) \le I(t_n)^2 + I(t_m)^2$$

$$\frac{1}{N^2}\sum_{n,m=0}^{\infty} I(t_n)I(t_m) \le \frac{1}{N^2}\sum_{n,m=0}^{\infty}\left(I(t_n)^2 + I(t_m)\right)^2 = \frac{1}{N}\sum_{n=0}^{\infty} I^2(t_n)$$

$\langle I\rangle^2 \le \langle I^2\rangle \quad \Rightarrow \quad \gamma^{(2)}(0) \ge 1$ (for classical light)

One can show similarly that

$0 \le \gamma^{(2)}(\tau) \le \infty$ and $\gamma^{(2)}(\tau) \le \gamma^{(2)}(0)$

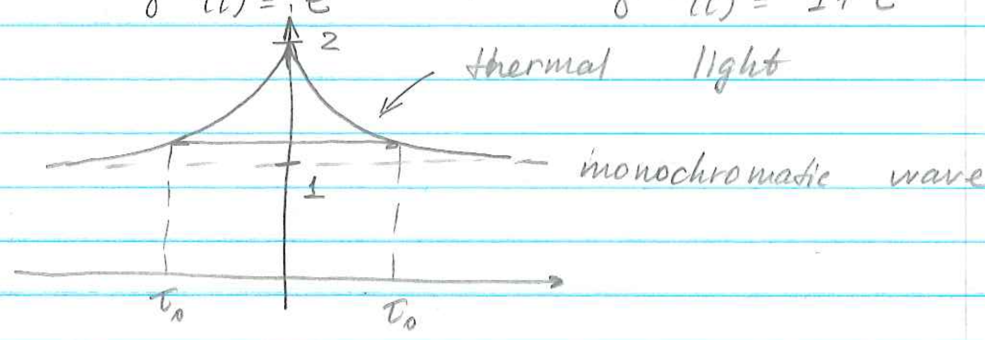
Coherent light $\gamma^{(2)}(t) = 1$

Thermal light (chaotic light)

(consists of many incoherent modes)

$$\gamma^{(2)}(\tau) = 1 + |\gamma^{(1)}(\tau)|^2$$

$\gamma^{(1)}(\tau) = e^{-|\tau|/\tau_0} \qquad \gamma^{(2)}(\tau) = 1 + e^{-2|\tau|/\tau_0}$



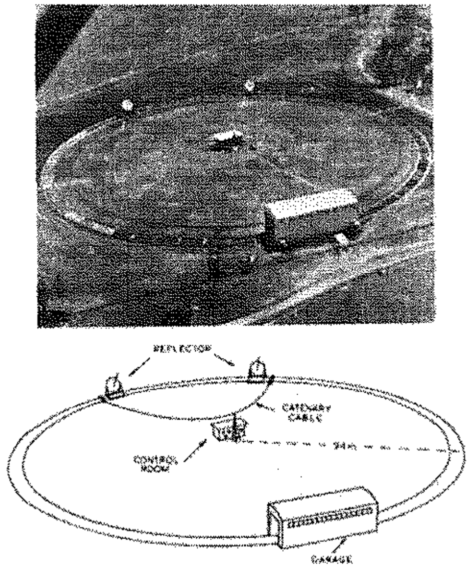


Figure 1. Aerial photo and illustration of the original HBT apparatus. They have been extracted from Ref.[1].

HBT interferometry, also known as two-identical-particle correlation, was idealized in the 1950's by Robert Hanbury-Brown, as a means to measuring stellar radii through the angle subtended by nearby stars, as seen from the Earth's surface.

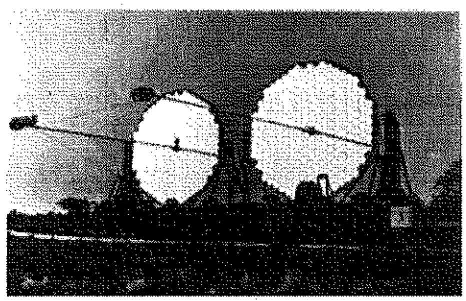


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

Before actually performing the experiment, Hanbury-Brown invited Richard Q. Twiss to develop the mathematical theory of intensity interference (second-order interference)[2]. A very interesting aspect of this experiment is that it was conceived by both physicists, who also built the apparatus themselves, made the experiment in Narrabri, Australia, and finally, analyzed the data. Nowadays, the experiments doing HBT at the RHIC/BNL accelerator have hundreds of participants. We could briefly summarize the experiment by informing that it consisted of two mirrors, each one focusing the light from a star onto a photo-multiplier tube. An essential ingredient of the device was the *correlator*, i.e., an electronic circuit that received the signals from both mirrors and multiplied them. As Hanbury-Brown himself described it, they " **... collected light as rain in a bucket ...** ", there was no need to form a conventional image: the (paraboloidal) telescopes used for radio-astronomy would be enough, but with light-reflecting surfaces. The necessary precision of the surfaces was governed by maximum permissible field of view. The draw-back they had to face in the first years was the skepticism of the community about the correctness of the results. Some scientists considered that the observation could not be real because it would violate Quantum Mechanics. In reality, in 1956, helped by Purcell [3], they managed to show that it was the other way round: not only the phenomenon existed, but it also followed from the fact that photons tended to arrive in pairs at the two correlators, as a consequence of Bose-Einstein statistics. A very interesting review about these early years was written by Gerson Goldhaber[1], one of the experimentalists responsible for discovering the identical particle correlation in the opposite realm of HBT: the microcosmos of high energy collisions.

1.2 GGLP

In 1959, Goldhaber, Goldhaber, Lee and Pais performed an experiment at the Bevalac/LBL, in Berkeley, CA, USA, aiming at the discovery of the r^0 resonance[4]. In the experiment, they considered $\bar{p}p$ collisions, at 1.05 GeV/c. They were searching for the resonance by means of the decay r^0 ® p^+p^-, by measuring the unlike pair, p^+p^-, mass-distribution and comparing it with the ones for like pairs, $p^{\pm}p^{\pm}$. Afterwards, they concluded that

Quantum coherence $\gamma^{(2)}$

First order

$$G^{(1)}(x_1,x_2) = \mathrm{Tr}\left(\rho\, \hat{E}^{(-)}(x_1)\, \hat{E}^{(+)}(x_2)\right)$$

Second order

$$G^{(2)}(x_1,x_2;x_2,x_1) = \mathrm{Tr}\left(\rho\, \hat{E}^{-}(x_1)\, \hat{E}^{-}(x_2)\, \hat{E}^{(+)}(x_2)\, \hat{E}^{(+)}(x_1)\right)$$

$$g^{(2)}(x_1,x_2;x_2,x_1) = \frac{G^{(2)}(x_1,x_2;x_2,x_1)}{G^{(1)}(x_1,x_1)\;G^{(1)}(x_2,x_2)}$$

For fixed detector position

$$g^{(2)}(\tau) = \frac{\langle \hat{E}^{(-)}(t)\,\hat{E}^{(-)}(t+\tau)\,\hat{E}^{(+)}(t+\tau)\,\hat{E}^{(+)}(t)\rangle}{\langle \hat{E}^{(-)}(t)\,\hat{E}^{(+)}(t)\rangle\langle \hat{E}^{(-)}(t+\tau)\,\hat{E}^{+}(t+\tau)\rangle}$$

For a single-mode field $E^{(+)} \propto \hat{a}$
$E^{(-)} \propto \hat{a}^{+}$

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^{+}\hat{a}^{+}\hat{a}\,\hat{a}\rangle}{\langle \hat{a}^{+}a\rangle^2} = \frac{\langle \hat{a}^{+}(-1+\hat{a}\hat{a}^{+})\hat{a}\rangle}{\langle \hat{a}^{+}\hat{a}\rangle^2} =$$

$$= \frac{\langle \hat{n}(\hat{n}-1)\rangle}{\langle \hat{n}\rangle^2} = 1 + \frac{\langle(\Delta n)^2\rangle - \langle \hat{n}\rangle}{\langle \hat{n}\rangle^2}$$

Coherent state $|\alpha\rangle$: $\langle n\rangle = |\alpha|^2 \quad \langle \Delta n\rangle = \sqrt{\langle n\rangle} = |\alpha|$

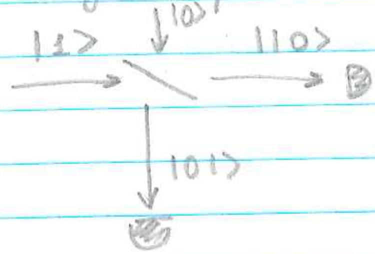
$$g^{(2)}(\tau) = 1 \qquad \text{(Poissonian statistics)}$$

Thermal state: $(\Delta n)^2 = \langle n\rangle^2 + \langle n\rangle$

$$g^{(2)}(\tau) = 2 \qquad \text{(photon bunching)}$$

Non-classical states: $g^{(2)}$ may be less than 1

Single photon



if $n = 1$

$g^{(2)} = 0$

photon antibunching

$$\boxed{\begin{array}{l} g^{(2)} = 1 + \dfrac{(\Delta n)^2 - \langle n \rangle}{\langle n \rangle^2} \\ \text{For any number state} \\ \quad \langle \Delta n \rangle = 0 \\ g^{(2)} = 1 - \dfrac{1}{n} < 1 \end{array}}$$

The states with $g^{(2)} < 1$ are "more ordered than random" — sup-Poissonian statistics

Number squeezed state (amplitude squeezing)

Clearly for any state with $(\Delta n)^2 < \langle n \rangle$
$g^{(2)} < 1$

Squeezed coherent state

$$|\alpha, \xi\rangle = \hat{D}(\alpha)\, S(\xi)\, |0\rangle$$

$$\langle n \rangle = |\alpha|^2 + \sinh^2 r$$

$$\langle \Delta n \rangle^2_{min} = |\alpha|^2 e^{-2r} + 2 \sinh^2 r \cosh^2 r$$

For large $|\alpha|$ $\quad \langle n \rangle \approx |\alpha|^2$
$\quad \langle \Delta n \rangle^2 \approx |\alpha|^2 e^{-2r}$

$$g^{(2)} = 1 + \frac{e^{-2r}-1}{|\alpha|^2} < 1$$

Not all non-classical states are $g^{(2)} < 1$

Squeezed vacuum

$\langle n \rangle = \sinh^2 r$

$\langle \Delta n \rangle = 2 \sin^2 r \cos^2 r$

$$g^{(2)} = 1 + \frac{2 \sinh^2 r \cosh^2 r - \sinh^4 r}{\sinh^4 r} =$$

$$= 1 + \frac{2\cosh^2 r - 1}{\sinh^2 r} = 1 + \frac{\cosh^2 r + \sinh^2 r}{\sinh^2 r} > 1$$

Squeezed vacuum statistics resembles a thermal state

$g^{(2)}$ can also serve to identify multi mode structure of the e-m field

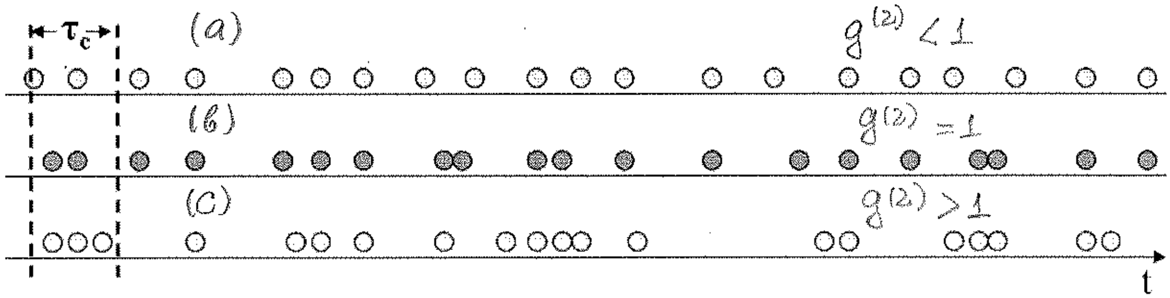
Very large number of modes (each one is coherent)

$$\langle a^+(t) a^+(t+\tau) \hat{a}(t+\tau) \hat{a}(t) \rangle =$$ (true for multimode (thermal) light)

$$= \langle \hat{a}^+(t) \hat{a}(t) \rangle \langle \hat{a}^+(t+\tau) \hat{a}(t+\tau) \rangle +$$
$$+ \langle \hat{a}^+(t) \hat{a}(t+\tau) \rangle \langle \hat{a}^+(t+\tau) \hat{a}(t) \rangle$$

$$\propto (1 + e^{-2|\tau|/\tau_c})$$

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2$$



Photon detections as a function of time for a) antibunched, b) random, and c) bunched light