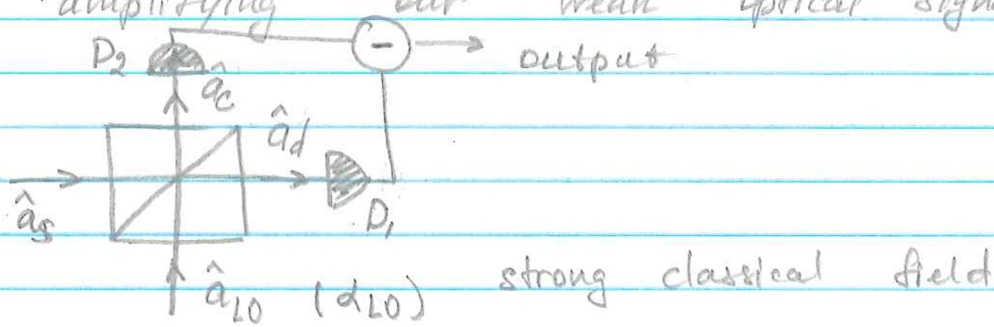
Balanced homodine detection

We want to take advantage of high quantum efficiency of the conventional detectors, by cleverly "amplifying" our weak optical signal



What we measure: $\Delta\hat n = \hat n_c - \hat n_d = \hat a_c^{+}\hat a_c - \hat a_d^{+}\hat a_d$

For an ideal 50/50 beam-splitters

$$\hat a_c = \frac{1}{\sqrt2}(\hat a_s + i\hat a_{LO})$$

$$\hat a_d = \frac{1}{\sqrt2}(i\hat a_s + \hat a_{LO}) = \frac{i}{\sqrt2}(\hat a_s - i\hat a_{LO})$$

$$\hat n_c = \hat a_c^{+}\hat a_c = \frac12(\hat a_s^{+} - i\hat a_{LO}^{+})(\hat a_s + i\hat a_{LO}) =$$
$$= \frac12(\underline{\hat a_{LO}^{+}\hat a_{LO}} + i\hat a_s^{+}\hat a_{LO} - i\hat a_{LO}^{+}\hat a_s + \hat a_s^{+}\hat a_s)$$

largest contribution, puts the signal above the dark noise

$$\hat n_d = \hat a_d^{+}\hat a_d = \frac12(\hat a_s^{+} + i\hat a_{LO}^{+})(\hat a_s - i\hat a_{LO}) =$$
$$= \frac12(\hat a_{LO}^{+}\hat a_{LO} - i\hat a_s^{+}\hat a_{LO} + i\hat a_{LO}^{+}\hat a_s + \hat a_s^{+}\hat a_s)$$

$$\Delta\hat n = i(\hat a_s^{+}\hat a_{LO} - \hat a_{LO}^{+}\hat a_s)$$

Detected current $I_{diff} \propto \langle \Delta n \rangle = i\langle \hat a_s^{+}\hat a_{LO} - \hat a_{LO}^{+}\hat a_s\rangle$

Again, here we explicitely assume that the local oscillator and the quantum signal are in the identical spatial and temporal modes, since we consider that after the beam-splitter we cannot distinguish which channel the photons came from

Normally, the local oscillator is a strong coherent state $|\alpha_{LO}\rangle$, and $\alpha_{LO} = |\alpha_{LO}| \cdot e^{i\chi}$

$$\langle a_{LO} \rangle = \alpha_{LO} \qquad \langle a_{LO}^{+} \rangle = \alpha_{LO}^{*}$$

$$\langle \Delta\hat{n} \rangle = i \langle \hat{a}_s^{+} \hat{a}_{LO} - \hat{a}_{LO}^{+} \hat{a}_s \rangle = \langle i |\alpha_{LO}| e^{i\chi} \hat{a}_s^{+} - i|\alpha_{LO}| e^{-i\chi} \hat{a}_s \rangle$$

$$= |\alpha_{LO}| \langle \underbrace{\hat{a}_s e^{-i\theta} + \hat{a}_s^{+} e^{i\theta}}_{2\hat{X}_\theta} \rangle \quad \text{if} \quad \theta = \chi + \pi/2$$

quadrature operator

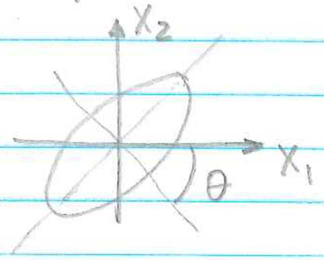
$$\langle \Delta\hat{n} \rangle = 2|\alpha_{LO}| \langle \hat{X}_\theta \rangle$$

Fluctuations of the differential photon flux

$$\langle \Delta\hat{n}^2 \rangle = 4|\alpha_{LO}|^2 \langle \hat{X}_\theta^2 \rangle$$

$$\Delta(\Delta n) = 4|\alpha_{LO}|^2 \langle \Delta\hat{X}_\theta^2 \rangle$$

Squeezed vacuum:



$\langle \hat{X}_1 \rangle = \langle \hat{X}_2 \rangle = 0$

For $\theta = 0$

$\langle \Delta \hat{X}_1 \rangle = \frac{1}{4} e^{-2r}$

$\langle \Delta \hat{X}_2 \rangle = \frac{1}{4} e^{2r}$

When analized using a balanced photodetector

$\Delta(\Delta n) = 4|\alpha_{LO}|^2 \langle \Delta \hat{X}_\theta^2 \rangle$

min: $|\alpha_{LO}|^2 e^{-2r}$

max: $|\alpha_{LO}|^2 e^{2r}$