Entanglement

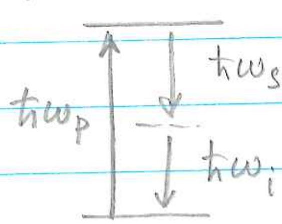
Two systems are placed in a quantum state that cannot be factorized into a product of individual systems.

Example: spontaneous emission

$$|\psi\rangle = e^{-\Gamma/2 t}|b,0\rangle + \sum_i |a,1_i\rangle c$$

For $t \lesssim 1/\Gamma$ the states of an atom and of a spontaneous photons are entangled (i.e. making a measurement on the atomic state will provide information about photons)

Most common soures of entangled photons – parametric down conversion



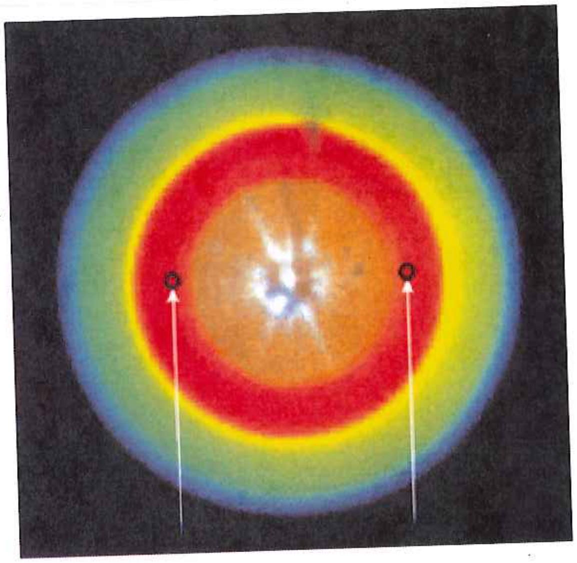
1 pump photon $(2\omega) \rightarrow$
$\rightarrow$ 1 signal + 1 idler photons (ω)

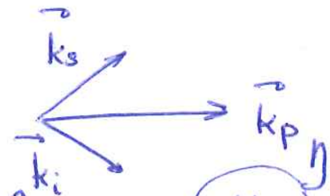
Nonlinear interaction

$$\hat{H}_{int} = \chi^{(2)}\, \hat{a}_p \hat{a}_s^+ \hat{a}_i^+ + c.c$$

$$\simeq \chi^{(2)} \alpha_p\, \hat{a}_s^+ \hat{a}_i^+$$ (for strong coherent pump laser

Energy conservation: $\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$
Momentum conservation $\hbar\vec{k}_p = \hbar\vec{k}_i + \hbar\vec{k}_s$
$\rightarrow$ Phase-matching conditions





$$\hat{H}_{int} = \chi^{(2)} \alpha_p \, \hat{a}_s^{+} \hat{a}_i^{+} + h.c$$

Initial state for signal & idler

$$|\psi_0\rangle = |0\rangle_s |0\rangle_i$$

$$|\psi(t)\rangle = e^{-i\hat{H}_{int} \cdot t/\hbar} \, |\psi(t=0)\rangle \approx$$

$$\approx \left(1 - \frac{i\chi\alpha_p}{\hbar} t \, \hat{a}_s^{+} \hat{a}_i^{+} + \frac{1}{2}\left(-\frac{i\chi\alpha_p}{\hbar}\right)^2 t^2 (\hat{a}_s^{+})^2 (\hat{a}_i^{+})^2 + \dots\right) |0\rangle_s |0_i\rangle =$$

$$\frac{\chi^{(2)}\alpha_p}{\hbar} t \ll 1$$

$$\approx |0\rangle_s |0_i\rangle + \overbrace{\left(\frac{i\chi^{(2)}\alpha_p}{\hbar} t\right)}^{\mu} |1\rangle_s |1\rangle_i + \dots$$

$$= |0\rangle_s |0_i\rangle - \mu \underbrace{|1\rangle_s |1\rangle_i}_{\text{correlated photon pair}}$$

For weak pumping

$$|\psi(t)\rangle = e^{-i\hat{H}_{int}t/\hbar}\,|\psi(0)\rangle \qquad |\psi(0)\rangle = |0\rangle_s|0\rangle_i$$

$$|\psi(t)\rangle \underset{\mu \ll 1}{\approx} \left(1 - i\hat{H}_{int}t/\hbar + \frac{1}{2}(-it\hat{H}_{int}/\hbar)^2 + \dots\right)|\psi(0)\rangle =$$

$$= \left(1 - \boxed{\frac{i\chi^{(2)}\alpha_p}{\hbar}}\, t\, \hat{a}_s^+ \hat{a}_i^+ + \dots\right)|0\rangle_s|0_i\rangle \approx |0\rangle_s|0_i\rangle - i\mu|1\rangle_s|1_i\rangle$$

Source of correlated photons, but not an entangled state

However, one can use this process to generate entanglement, if we consider two possible polarizations

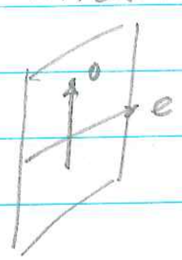
Because of the phase-matching conditions

$$\hbar\frac{n_p\omega_p}{c}\vec{e}_p = \hbar\frac{n_i\omega_i}{c}\vec{e}_i + \hbar\frac{n_s\cdot\omega_s}{c}\cdot\vec{e}_s$$

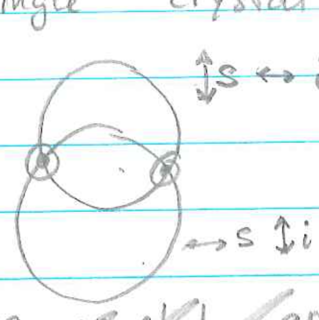
if the refractive indices n_i and n_s are different for different polarizations, they will phase-match at different angles.

Birefringent nonlinear crystals

$n_e \neq n_o$



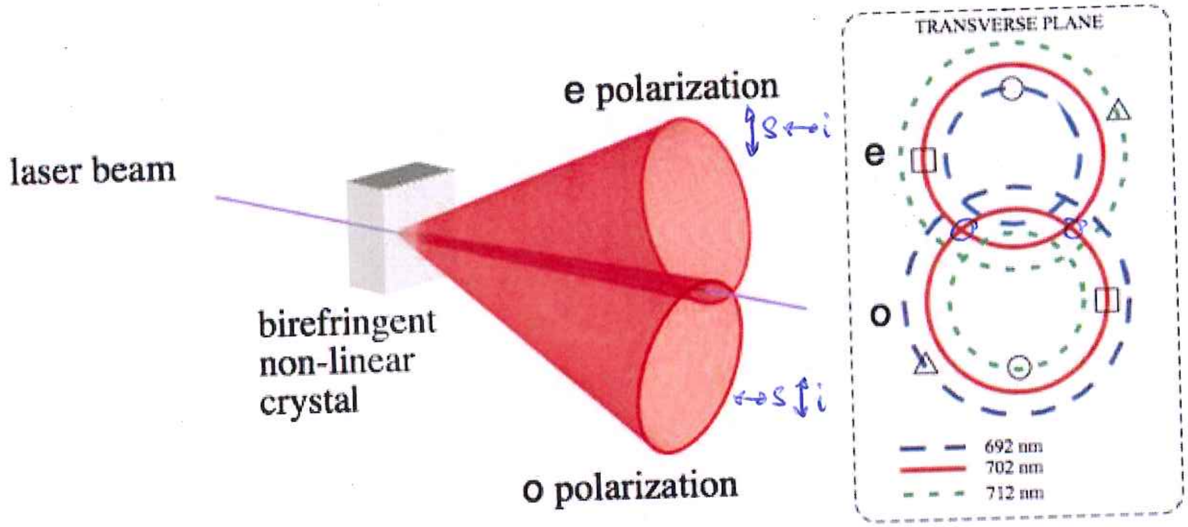
single crystal



Type II down-conversion

~~Alternatively – two crystal arrangement~~

~~two overlapping cones with orthogonal polarizations~~



$$\hat{H}_{int} = \hbar\chi\left(\hat{a}^{\dagger}_{V_s}\hat{a}^{\dagger}_{H_i} + \hat{a}^{\dagger}_{H_s}\hat{a}^{\dagger}_{V_i}\right) + H.c$$

$$|\psi_0\rangle = |0\rangle_{sV}|0\rangle_{sH}|0\rangle_{iV}|0\rangle_{iH}$$

$$\cancel{H_i}\quad |\psi(t)\rangle = |0\rangle_{sV}|0\rangle_{sH}|0_{iV}\rangle|0_{iH}\rangle + \mu\frac{1}{\sqrt{2}}\Big(|1\rangle_{sV}|0\rangle_{sH}|0\rangle_{iV}|1\rangle_{iH} + |0\rangle_{sV}|1\rangle_{sH}|1\rangle_{iV}|0\rangle_{iH}\Big)$$

$|0\rangle_{V}|1\rangle_{H} = |H\rangle$

$|1\rangle_{V}|0\rangle_{H} = |V\rangle$

$$|\psi(t)\rangle = |0\rangle_{vac} + \frac{\mu}{\sqrt{2}}\underbrace{\left(|V\rangle_s|H\rangle_i + |H\rangle_s|V_i\rangle\right)}_{\text{entangled state}}$$

$$\hat{H}^{(2)}_{int} = \hbar g\left(\hat{a}^{\dagger}_{Vs}\,\hat{a}_{Hi} + \hat{a}^{\dagger}_{Hs}\,\hat{a}^{\dagger}_{Vi}\right) + H.c.$$

$$|\psi^{(2)}(t)\rangle \approx |0\rangle_{Vs}|0\rangle_{Hs}|0\rangle_{Vi}|0\rangle_{Hi} -$$

$$- i\mu\,\frac{1}{\sqrt{2}}\left(\underbrace{|1\rangle_{Vs}|0\rangle_{HS}}_{|V\rangle_s}\,\underbrace{|0\rangle_{Vi}|1\rangle_{Hi}}_{|H\rangle_i} + \underbrace{|0\rangle_{Vs}|1\rangle_{HS}}_{|H\rangle_s}\,\underbrace{|1\rangle_{Vi}|1\rangle_{Vs}}_{|V\rangle_i}\right)$$

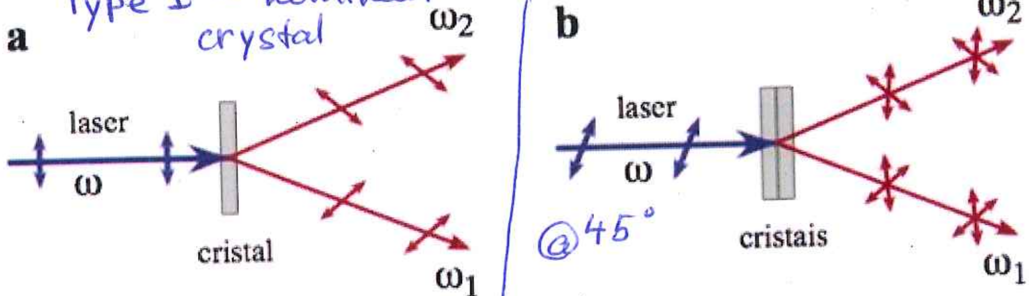
$$|\psi^{(2)}(t)\rangle \approx |0\rangle_s|0\rangle_i - \frac{i\mu}{\sqrt{2}}\left(\underbrace{|V\rangle_s|H\rangle_i + |H\rangle_s|V\rangle_i}_{\text{polarization-entangled two-photon state}}\right)$$

One of Bell states

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|V\rangle_s|H\rangle_i \pm |H_s\rangle|V_i\rangle\right)$$

$$|\varphi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_s|H\rangle_i \pm |V\rangle_s|V\rangle_i\right)$$

Type I - nonlinear crystal



@45°

$$|H\rangle_p \rightarrow |V\rangle_s |V\rangle_i$$

$$\hat{H}_{int} = \hbar g\, a^{\dagger}_{Vs} a^{\dagger}_{Vi} + h.c.$$

$$\hat{H}_{int} = \hbar g \left(\hat{a}^{\dagger}_{Vs} \hat{a}^{\dagger}_{Vi} + \hat{a}^{\dagger}_{Hs} \hat{a}^{\dagger}_{Hi} \right) + h.c.$$

$$|\psi(t)\rangle = |vac\rangle + \frac{1}{\sqrt{2}} \mu \left(|V\rangle_s |V\rangle_I + |H\rangle_s |H\rangle_I \right)$$

entangled state

Bell states

eigen-basis for two-particles entangle states are

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}} \left(|V\rangle_s |H\rangle_I \pm |H\rangle_s |V\rangle_I \right)$$

$$|\varphi^{\pm}\rangle = \frac{1}{\sqrt{2}} \left(|V\rangle_s |V\rangle_I \pm |H\rangle_s |H\rangle_I \right)$$