Quantum Rabi flopping

$$|\psi(t)\rangle = C_a(t)\,|a, n+1\rangle + C_b(t)\,|b, n\rangle$$

let $c_b(t=0) = 1 \qquad c_a(t=0) = 1$

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \qquad \text{gives common phase factor}$$

$$i\hbar \begin{pmatrix} \dot{c}_b \\ \dot{c}_a \end{pmatrix} = \cancel{\hbar\omega\left(n+\tfrac{1}{2}\right)\begin{pmatrix} c_b \\ c_a \end{pmatrix}} + \frac{1}{2}\hbar \begin{pmatrix} \overset{=0}{\cancel{\Delta}} & 2g\sqrt{n+1} \\ 2g\sqrt{n+1} & \underset{=0}{\cancel{\Delta}} \end{pmatrix} \begin{pmatrix} c_b \\ c_a \end{pmatrix}$$

for simplicity

$$i\hbar\, \dot{c}_b = \hbar g\sqrt{n+1}\; c_a$$
$$i\hbar\, \dot{c}_a = \hbar g\sqrt{n+1}\; c_b$$

$$\ddot{c}_b + g^2(n+1)\, c_b = 0$$

$$c_b = \cos\left(g\sqrt{n+1}\, t\right) \qquad P_b = \cos^2\left(g\sqrt{n+1}\, t\right)$$
$$c_a = -i \sin\left(g\sqrt{n+1}\, t\right) \qquad P_a = \sin^2\left(g\sqrt{n+1}\, t\right)$$

$$|\psi(t)\rangle = \cos\left(g\sqrt{n+1}\, t\right)|b, n\rangle - i\sin\left(g\sqrt{n+1}\, t\right)|a, n+1\rangle$$

Quantum Rabi fluctuations

notice that even if $n=0$ (vacuum)
there still will be floppings
with frequency $2g$
(vacuum Rabi flopping)

So, a Fock state with a fixed number of photons $|n\rangle$ behaves very similarly to a classical Rabi flopping.

What about a coherent state?

Initially

$$|\psi_{atom}\rangle_0 = c_a|a\rangle + c_b|a\rangle$$

$$|\psi_{light}\rangle_0 = \sum_{n=0}^{\infty} c_n |n\rangle \qquad c_n = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}}$$

for a coherent state

$$|\psi(t=0)\rangle = |\psi_{atom}\rangle_0 \, |\psi_{light}\rangle_0$$

As we discussed before, light-atom interaction couples states $|a, n+1\rangle$ and $|b, n\rangle$ for all n present.

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} \Big\{ \left[c_b c_n \cos(gt\sqrt{n+1}) - i c_a c_{n+1} \sin(gt\sqrt{n+1}) \right] \times |b\rangle$$

$$+ \left[-i c_b c_{n-1} \sin(gt\sqrt{n}) + c_a c_n \cos(gt\sqrt{n}) \right] |a\rangle \Big\} |n\rangle$$

For $c_b = 1$ (we start with an atom in the excited state)

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} \left\{ c_n \cos(gt\sqrt{n+1}) |b\rangle - i c_{n-1} \sin(gt\sqrt{n}) |a\rangle \right\} |n\rangle$$

$$|\psi_a(t)\rangle = \sum_{n=0}^{\infty} c_n \sin(gt\sqrt{n+1}) |n+1\rangle \qquad \text{ground}$$

$$|\psi_b(t)\rangle = \sum_{n=0}^{\infty} c_n \cos(gt\sqrt{n+1}) |n\rangle \qquad \text{excited}$$

Average atomic inversion

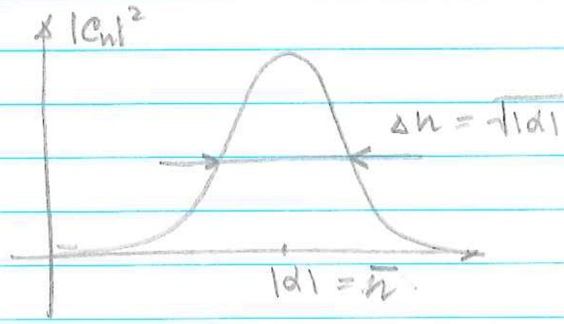
$$\langle \psi(t) | \hat{\sigma}_3 | \psi(t) \rangle = \langle \psi(t) | \, |b\rangle\langle b| - |a\rangle\langle a| \, | \psi(t) \rangle =$$

$$= \langle \psi_b | \psi_b \rangle - \langle \psi_a | \psi_a \rangle = \sum_{n=0}^{\infty} |C_n|^2 \{ (\cos^2 gt\sqrt{n+1}) -$$

$$- \sin^2(gt\sqrt{n+1}) \} = \sum_{n=0}^{\infty} |C_n|^2 \cos 2gt\sqrt{n+1} =$$

$$= e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^n}{n!} \cos\left(2gt\sqrt{n+1}\right)$$

The output is a combinations of many sine waves with somewhat different periods $\longrightarrow$ no clear Rabi fluctuations



Main contributing components lie between frequencies $2g\sqrt{\bar{n} - \Delta n}$ and $2g\sqrt{\bar{n} + \Delta n}$

Corresponding phase spread

$$2gt_c(\sqrt{\bar{n} + \Delta n} - \sqrt{\bar{n} - \Delta n}) \approx 2gt_c\sqrt{\bar{n}} \left(\left(1 + \frac{\Delta n}{2\bar{n}}\right) - \left(1 - \frac{\Delta n}{2\bar{n}}\right) \right)$$

$$\approx 2gt_c \frac{\Delta n}{2\sqrt{\bar{n}}} \approx 1 \qquad \Rightarrow gt_c \approx 1$$

$t_c \approx 1/g$ depends only on coupling strength

However we can also expect to see a revival of Rabi oscillations if

$$\left(g\sqrt{\bar{n}+1} - g\sqrt{\bar{n}}\right) t_R = 2\pi k \qquad (k=0,1,2...$$

$$g\sqrt{\bar{n}}\left(\cancel{1}+\frac{1}{2\bar{n}} - \cancel{1}\right) t_R = 2\pi \quad \text{(k=1 for the first occurance)}$$

$$g\frac{1}{2\sqrt{\bar{n}}} t_R = 2\pi$$

$$t_R = \frac{4\pi\sqrt{\bar{n}}}{g}$$

The revival is never "complete" since the frequencies $\{g\sqrt{n}\}$ are not truely equidistant.

Why a coherent state is less "classical" than a number state?

Clear Rabi flopping require knowledge of precise intensity. Coherent state, as a minimum uncertainty state, has certain spread in its intensity distribution, that leads to the Rabi flopping diffusion