

Physics 786, Spring 2023

Problem Set 4 Due Friday, March 3, 2023.

Short homework this week.

1. *Geodesics on the 2-sphere*

In spherical coordinates, the length element on the 2-sphere of radius R takes the form

$$ds^2 = R^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

a) With $x^1 = \theta$ and $x^2 = \phi$, the metric $g_{ij} = g_{ji}$ is defined such that $ds^2 = g_{ij} dx^i dx^j$, summed over i and j . What are the components of g_{ij} , written as a 2×2 matrix?

b) Find the nonvanishing components of the connection

$$\Gamma_{jk}^i = \frac{1}{2} g^{im} \left(\frac{\partial g_{mj}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^m} \right).$$

c) Consider a path parametrized by a parameter t . The paths of shortest distance satisfy the geodesic equation:

$$\frac{d^2 x^i}{dt^2} + \Gamma_{jk}^i \frac{dx^j}{dt} \frac{dx^k}{dt} = 0.$$

Show that arcs along the equator $\theta = \pi/2$ are geodesics on the 2-sphere.