

Physics 786, Spring 2023

Problem Set 10, Due Friday, April 28.

A complete draft of your paper is due this Friday. Let me know if you have questions about your paper topic as you are writing it.

1. *Newtonian Stars*

Consider static, spherically symmetric solutions to Einstein's equations for a fluid with density $\rho(r)$ and pressure $p(r)$, with metric of the form,

$$ds^2 = -e^{2\phi(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2).$$

Assume the fluid is nonrelativistic, and consider the nonrelativistic limit of Einstein's equations for this system. Assume $\phi(0) = \lambda(0) = 0$, and

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right)$$

for $r \leq R$, and $\rho(r) = 0$ for $r > R$.

a) Find the spacetime metric for $r < R$ and $r > R$. The metric should be continuous across $r = R$.

b) Find the pressure $p(r)$ in the star such that $p(R) = 0$.

2. *Gravitational Radiation from Binary Black Holes*

Suppose two black holes in a binary black hole system each have 30 solar masses, with orbital period beginning at $T = 0.01$ s when the radius of the circular orbit about the center of mass is $R = 140$ km.

Treating the system in a Newtonian approximation, find the power radiated in gravitational radiation in Watts.