Physics 772, Spring 2009
Problem Set 6 Due Thursday, April 23.

1. *Standard Model Gauge Anomalies*

Check that the SU(3)×SU(2)×U(1) gauge invariance of the Standard Model is anomaly free.

The following problems are based on Andre de Gouvea's neutrino review:

2. *Phases in the PMNS Matrix and CP violation*

With three neutrino flavors, the $\nu_\mu \to \nu_e$ oscillation probablility can be written,

$$P_{\nu_\mu \to \nu_e} = \sum_{i,j=1}^{3} U_{ei}^* U_{\mu i} U_{ej} U_{\mu j}^* \exp\left(-i\frac{(m_i^2 - m_j^2)L}{2E}\right). \tag{1}$$

The probability for $\nu_e \to \nu_\mu$ oscillations takes the same form, except $\mu \leftrightarrow e$.

The probability for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations takes the same form as (1), except $U \leftrightarrow U^*$.

a) Calculate $P_{\nu_\mu \to \nu_e} - P_{\nu_e \to \nu_\mu}$ and show that time reversal invariance is not necessarily conserved unless U is a real matrix.

b) Calculate $P_{\nu_\mu \to \nu_e} - P_{\overline{\nu}_\mu \to \overline{\nu}_e}$ and compare with your result of part (a). Can there be CP violation in ν_μ–ν_e neutrino mixing if there are only Majorana phases in the PMNS matrix, *i.e.* $\delta = 0$?

c) Show that CPT is conserved, *i.e.* $P_{\nu_\mu \to \nu_e} = P_{\overline{\nu}_e \to \overline{\nu}_\mu}$.

3. *Day-Night Effect*

In class we spoke about matter effects on neutrino oscillation probablilities due to charged current interactions in a background of electrons. As a result of matter effects, the 2-component oscillation frequency becomes,

$$\Delta_M = \sqrt{\Delta^2 \sin^2 2\theta + (\Delta \cos 2\theta - \sqrt{2} G_F N_e)^2},$$

where $\Delta = \Delta m^2/(2E)$ and N_e is the background electron density. The matter mixing angle obeys,

$$\Delta_M \sin 2\theta_M = \Delta \sin 2\theta,$$

$$\Delta_M \cos 2\theta_M = \Delta \cos 2\theta - \sqrt{2} G_F N_e.$$

a) Assume that a ^{8}B neutrino reaches the earth as a $|\nu_2\rangle$ eigenstate, where the vacuum mixing angle is defined such that,

$$|\nu_e\rangle = \cos\theta \, |\nu_1\rangle + \sin\theta \, |\nu_2\rangle .$$

The neutrino then propagates a distance L through the Earth before reaching the detector. Assume that the electron number density in the neutrino's path is constant. Compute the probability that the neutrino is detected as an electron-type neutrino.

b) Assume $\sqrt{2} G_F N_e$= 1.5×10^{-7} eV^2/MeV, Δm^2=10^{-5} eV^2, $\sin^2\theta = 0.3$, and $E = 8$ MeV. Assume that at night the neutrino passes through a distance L =3000 km of Earth, whereas during the day the neutrino is detected before passing through any matter. Compute $P^{\text{night}}_{\nu_2\to\nu_e} - P^{\text{day}}_{\nu_2\to\nu_e}$, where ν_2 is the heavier neutrino mass eigenstate.