**Physics 772, Spring 2009**
**Problem Set 4** Due Tuesday, March 31.

1. *Top quark decays*

a) The top quark decays almost exclusively to a bottom quark plus a W boson. Approximating $|V_{tb}| = 1$, calculate the decay rate $\Gamma(t \to W^+ b)$. Since the $b$ quark is much lighter than the $t$ quark, you may approximate $m_b = 0$ for this calculation. You should sum over final spins and average over initial spins, and use

$$\sum_{r=1}^{3} \left(\epsilon_\mu^{r\,*}(k)\epsilon_\nu^r(k)\right) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2},$$

where $\epsilon_\mu^r(k)$ is the polarization of the W boson with helicity labeled by $r$ and momentum $k$.

b) Not assuming $|V_{tb}| = 1$, calculate the tree-level branching ratio for this decay in terms of the CKM matrix elements,

$$BR(t \to W^+ b) = \frac{\Gamma(t \to W^+ b)}{\Gamma(t \to W^+ q)},$$

where in the denominator you should sum over the decay widths to all possible quarks. If possible, simplify your expression using unitarity of the CKM matrix.

c) A global fit to the *magnitude* of the CKM matrix elements quoted in the 2008 Review of Particle Physics is:

$$V_{CKM} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.0023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

Estimate the branching ratio calculated in part (b).

d) The experimental values in $V_{CKM}$ quoted above make use of unitarity in the fit. By unitarity the diagonal elements of $V_{CKM}V_{CKM}^\dagger$ and $V_{CKM}^\dagger V_{CKM}$ should each be 1. Check that the experimental data quoted above agree with these unitarity constraints to a part in $10^5$.

2. *Polarization Asymmetry*

Consider the decay of a Z boson to pairs of fermions, $Z \to f\overline{f}$. The decay rate to $f_L\overline{f_L}$ differs from that to $f_R\overline{f_R}$ because of the chiral nature of the electroweak interactions. Although chirality eigenstates are not equivalent to helicity eigenstates, for fast-moving fermions they are nearly equivalent. Hence, the polarization asymmetry in Z decays to light fermions can be approximated as the asymmetry in decays to left vs. right-handed fermions.

a) Calculate the left-right asymmetry (at tree level) for $Z$ decays to a particular fermion $f$, defined by

$$A_{LR}^f = \frac{\Gamma(Z^0 \to f_L\overline{f_L}) - \Gamma(Z^0 \to f_R\overline{f_R})}{\Gamma(Z^0 \to f_L\overline{f_L}) + \Gamma(Z^0 \to f_R\overline{f_R})},$$

as a function of the fermion's electric charge and $\sin^2\theta_W$. You may neglect the fermion mass.

b) Using the experimental value $\sin^2\theta_W = 0.23$ calculate the left-right asymmetry for charged leptons. Compare with the experimental value for the electron polarization asymmetry quoted in the 2008 Review of Particle Physics, $A_{LR}^e = 0.15138 \pm 0.00216$. Also calculate the polarization asymmetry for down type quarks.