Physics 722, Spring 2021
Problem Set 3, due Thursday, February 25.

1. *Fermion self energy*

Consider the theory of a fermion $\psi(x)$ with mass m, Yukawa coupled to a real scalar field $\phi(x)$ with mass μ:

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - g\overline{\psi}\psi\phi - \frac{\lambda_3}{3!}\phi^3 - \frac{\lambda_4}{4!}\phi^4 + \mathcal{L}_{\rm CT}.$$

a) Calculate the one-loop renormalized fermion self energy $\widetilde{\Sigma}(p\!\!\!/)$. The renormalized self energy should satisfy $\widetilde{\Sigma}(M) = 0$ and $d\widetilde{\Sigma}/dp\!\!\!/\,|_{p\!\!\!/=M} = 0$. Use a hard momentum cutoff to regularize any divergent integrals appearing at intermediate stages of the calculation, and check that those divergences can be cancelled by counterterms in the renormalization procedure. Your result should be left in terms of integral(s) over a single Feynman parameter.

b)Does $\widetilde{\Sigma}(p\!\!\!/)$ have a branch cut? If so, what is the physical interpretation of the value of p^2 at the branch point (not at infinity)?