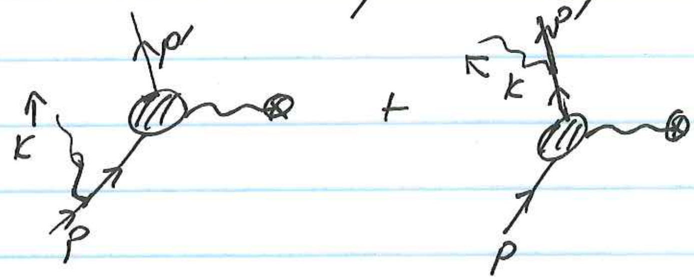
Bremsstrahlung – the radiation of photons by a charged particle when it interacts.

Consider the interaction of an electron w/ an external electromagnetic field



Call M_0 the part of the amplitude coming from the interaction w/ the external field. Then the amplitude for the Bremsstrahlung process is,

$$i\mathcal{M} = \bar{u}(p')\Big[M_0(p', p-k) \frac{i(\not{p}-\not{k}+m)}{(p-k)^2-m^2+i\epsilon} e\gamma^\mu \epsilon^*_\mu(k)$$

$$+ e\gamma^\mu \epsilon^*_\mu(k) \frac{i(\not{p}'+\not{k}+m)}{(p'+k)^2-m^2+i\epsilon} M_0(p'+k, p)\Big] u(p)$$

If the emitted photon is soft, then $M_0(p', p-k) \approx M_0(p', p)$ and $M_0(p'+k, p) \approx M_0(p', p)$.

The numerators are both simplified when $k \approx 0$:

$$(\not{p}+m)\gamma^\mu u(p) = \Big[2p^\mu + \gamma^\mu \underbrace{(-\not{p}+m)}_{0}\Big] u(p)$$
$$= 2p^\mu u(p)$$

$$\bar{u}(p')\gamma^\mu(\not{p}'+m) = \bar{u}(p')\left[2p'^\mu + \underbrace{(-\not{p}'+m)}_{0}\gamma^\mu\right]$$

$$= \bar{u}(p')\cdot 2p'^\mu$$

The denominators become: $(p-k)^2 - m^2 \approx -2p\cdot k$

$$(p'+k)^2 - m^2 \approx +2p'\cdot k$$

So, $$iM \approx \underbrace{\bar{u}(p')M_0(p',p)u(p)}_{\text{Amplitude for elastic scattering w/o bremsstrahlung.}}\left[e\left(\frac{p'\cdot\epsilon^*}{p'\cdot k} - \frac{p\cdot\epsilon^*}{p\cdot k}\right)\right]$$

Summing over polarization states of the photon, the differential cross section gets a factor,

$$d\sigma(p\to p'+\gamma) = d\sigma(p\to p')\int\frac{d^3k}{(2\pi)^3 2\omega_k}\sum_{\lambda=1,2} e^2\left|\frac{p'\cdot\epsilon^{(\lambda)}}{p'\cdot k} - \frac{p\cdot\epsilon^{(\lambda)}}{p\cdot k}\right|^2$$

$$\omega_k = k$$

For the photon polarization sum we use $\sum_\lambda \epsilon_\mu^{(\lambda)}\epsilon_\nu^{(\lambda)*} \to -g_{\mu\nu}$

$$d\sigma(p\to p'+\gamma) = d\sigma(p\to p')\int\frac{d^3k}{(2\pi)^3 2\omega_k} e^2(-g_{\mu\nu})\left(\frac{p'^\mu}{p'\cdot k} - \frac{p^\mu}{p\cdot k}\right)\left(\frac{p'^\nu}{p'\cdot k} - \frac{p^\nu}{p\cdot k}\right)$$

$$= d\sigma(p\to p')\int\frac{d^3k}{(2\pi)^3 2\omega_k} e^2\left(\frac{2p\cdot p'}{(k\cdot p')(k\cdot p)} - \frac{m^2}{(p'\cdot k)^2} - \frac{m^2}{(p\cdot k)^2}\right)$$

The angular integral can be done by choosing a nice frame, like $p^0 = p'^0$. The resulting integral is of the form,

$$d\sigma(p \to p' + \gamma) = d\sigma(p \to p') \int dK \, \frac{e^2}{4\pi^2} \frac{1}{K} \underbrace{I(\vec{p}, \vec{p}')}_{\text{Independent of } K}$$

The K integral gives a log from the region where the emitted photon has $K \approx 0$.
If the photon had a small mass we could approximate the integral by replacing the lower bound by μ.

Since we are assuming $K \ll |\vec{p}' - \vec{p}| = |\vec{q}|$, we cutoff the integral at some $K \sim |\vec{q}|$.

Hence, $d\sigma(p \to p' + \gamma) \simeq d\sigma(p \to p') \frac{e^2}{8\pi^2} \log\left(\frac{-q^2}{\mu^2}\right) I(\vec{p}, \vec{p}')$

The factor $I(\vec{p}, \vec{p}')$ can be evaluated as $-q^2 \to \infty$, and gives $I(\vec{p}, \vec{p}') \to 2 \log\left(-\frac{q^2}{m^2}\right)$.

So, $\boxed{d\sigma(p \to p' + \gamma) \approx d\sigma(p \to p') \frac{e^2}{4\pi^2} \log\left(\frac{-q^2}{\mu^2}\right) \log\left(\frac{-q^2}{m^2}\right)}$

$-q^2 \gg m^2$.

We have recovered the double log that appeared in the IR divergent contribution to the cross section w/o bremsstrahlung from $F_1(q^2)$, but w/ the opposite sign.

As $\mu \to 0$ the divergent parts exactly cancel.

The probability of a scattering event with or without an undetected photon w/ energy $< E_{min}$ is proportional to,

$$\left.\frac{d\sigma}{d\Omega}\right|_{measured} = \frac{d\sigma}{d\Omega}(p \to p') + \frac{d\sigma}{d\Omega}(p \to p' + \gamma(k < E_{min}))$$

$$\to \left.\frac{d\sigma}{d\Omega}(p \to p')\right|_{\substack{tree\\level}} \left[1 + \frac{e^2}{8\pi^2} I(\vec{p}, \vec{p}\,') \log \frac{E_{min}^2}{f(m^2 \text{ and } -q^2)}\right]$$

Requires more detailed calculation

$$\xrightarrow{-q^2 \gg m^2} \left.\frac{d\sigma}{d\Omega}(p \to p')\right|_{\substack{tree\\level}} \left[1 - \frac{e^2}{4\pi^2} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{E_{min}^2}\right) + \mathcal{O}(e^4)\right]$$

The final result depends on the experimental conditions through E_{min}, but not on the fake photon mass μ.

By carefully summing over the cross sections for bremsstrahlung of arbitrary numbers of photons, one obtains for $-q^2 \gg m^2$,

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{measured} = \left.\frac{d\sigma}{d\Omega}\right|_{\substack{tree\\level}} \underbrace{\left[\exp\left(-\frac{e^2}{8\pi^2} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{E_{min}^2}\right)\right)\right]^2}_{\text{"Sudakov form factor"}}}$$